SAMPLE PAPER 1

1. **(b)** Since, the graph of $y = f(x)$ is a parabola, therefore $f(x)$ is quadratic.

2. **(b)** The pair of linear equations in two variables is also known as simultaneous equations.

3. **(a)** All isosceles triangles are not similar.

4. **(b)** Quantity of juice = Volume of cylinder – volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \pi \times 3 \times 3 \times 15 - \frac{2}{3}\pi \times 3 \times 3 \times 3 = 135\pi - 18\pi = 117\pi$$

5. **(c)** $\cos A = \frac{3}{5} \Rightarrow \sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

Consider $9\cot^2 A - 1 = \frac{9\cos^2 A}{\sin^2 A} - 1 = \frac{9\cos^2 A - \sin^2 A}{\sin^2 A} = \frac{9\left(\frac{9}{25}\right)-\left(\frac{16}{25}\right)}{\frac{16}{25}} = \frac{(81-16)}{25} \times \frac{25}{16} = \frac{65}{16}$

6. **(c)**

7. **(d)** The arithmetic sequence of terms is as follows:

$t_1, t_2, t_3, \ldots\ldots\ldots\ldots t_{n-1}, t_n$

Here, sum to n terms = Sn

And, sum to (n – 1) terms = S_{n-1}

So,

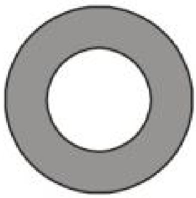
$S_n = t_1 + t_2 + t_3 + \ldots\ldots\ldots + t_{n-1} + t_n$ ... (i)

$S_{n-1} = t_1 + t_2 + t_3 + \ldots\ldots + t_{n-2} + t_{n-1}$ ... (ii)

Subtracting (ii) from (i), we get : $S_n - S_{n-1} = t_n$.

8. **(c)** Let the radius of inner circle be r cm.

Then, its circumference = $(2\pi r)$ cm.



$\therefore \quad 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = \left(88 \times \frac{7}{44}\right) = 14\text{cm}$.

$\therefore$ Radius of the inner circle is, r = 14 cm.

Let the radius of the outer circle be R cm.

Then, area of the ring = $(\pi R^2 - \pi r^2)$ cm². = $\left(\frac{22}{7}R^2 - 616\right)$ cm²

$\therefore \frac{22}{7}R^2 - 616 = 346.5 \Rightarrow \frac{22}{7}R^2 = 962.5$

$\Rightarrow R^2 = \left(962.5 \times \frac{7}{22}\right) = 306.25 \Rightarrow R = \sqrt{306.25} = 17.5\text{cm}$

Hence, the radius of the outer circle is 17.5 cm.

9. **(c)** Let the diameter of the sphere be 'd'

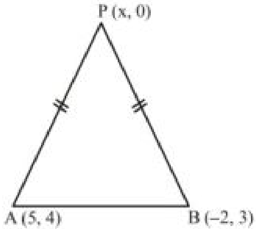
then radius of the sphere is $\frac{d}{2}$

When the diameter is doubled, then the new radius is $\frac{2d}{2} = d$

So,

$$\frac{\text{Surface Area of new sphere}}{\text{Surface Area of original sphere}} = \frac{4\pi d^2}{4\pi\left(\frac{d}{2}\right)^2} = \frac{1}{114} = 4$$

10. (a) Since, the required point (say P) is on the x-axis, its ordinate will be zero. Let the abscissa of the point be x.



Therefore, coordinates of the point P are (x, 0).

Let A and B denote the points (5, 4) and (–2, 3) respectively.

Given that AP = BP, we have

$AP^2 = BP^2$

i.e. $(x-5)^2 + (0-4)^2 = (x+2)^2 + (0-3)^2$

$\Rightarrow x = 2$

11. **Answer :** An irrational number.

12. Let $x = \sqrt{6+\sqrt{6+\sqrt{6+}}}$ ......

$\Rightarrow x = \sqrt{6+x}$

Squaring both sides

$\Rightarrow \quad x^2 = 6 + x \Rightarrow x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0 \Rightarrow x = -2, 3.$

Since, square root of any number cannot be negative.

$\therefore x = 3$

Answer : 3

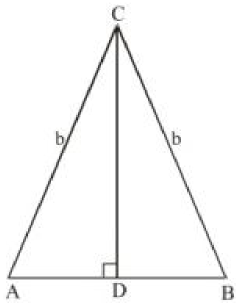
13. Required term $= l - (n-1)\,d = 253 - (20-1)(5) = 253 - 95 = 158$

Answer : 158

OR

Answer : $4\sqrt{2}$

14. Let ABC be an isosceles triangle, where base AB = a and equal sides $AC = BC = b$. Let CD be the perpendicular on AB.



So, $AD = DB = \frac{1}{2}AB = \frac{a}{2}$

Altitude, CD = height of the ΔABC and is given by

$h = \sqrt{AC^2 - AD^2} \Rightarrow h = \frac{1}{2}\sqrt{4b^2 - a^2}$

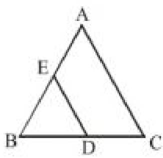
Area of the $\Delta ABC = \frac{1}{2}$ base × altitude

$= \frac{1}{2} \times a \times \frac{1}{2}\sqrt{4b^2 - a^2} = \frac{a}{4}\sqrt{4b^2 - a^2}$.

Answer : $\frac{a}{4}\sqrt{4b^2 - a^2}$

15. All equilateral triangles are similar

$\therefore \Delta ABC \sim \Delta EBD$



$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{BC^2}{BD^2} = \frac{(2BD)^2}{BD^2} = \frac{4}{1}$$

$\Rightarrow$ Area (ΔABC) : Area $(\Delta BDE) = 4 : 1$

Answer : 4 : 1

16. $x^2 - 3x + 2 = 0$

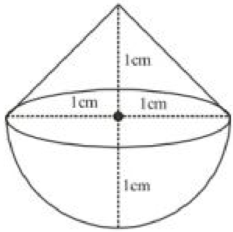
On comparing with $ax^2 + bx + c = 0$, we get $a = 1, b = -3, c = 2$

$D = b^2 - 4ac = (-3)^2 - 4(1)(2) = 1 \Rightarrow \sqrt{D} = 1$

The two roots are given by $\frac{-b \pm \sqrt{D}}{2a}$, *i.e.*, $\frac{3 \pm 1}{2} = \frac{4}{2}$ and $\frac{2}{2}$ [1 Mark]

Hence, the two roots are 1 and 2.

17. Radius of the hemisphere = 1 cm



Then, volume of the hemispherical part $= \frac{2\pi}{3} r^3$

$= \frac{2\pi}{3} \times (1)^3 \text{ cm}^3 = \frac{2\pi}{3} \text{cm}^3$

Radius and vertical height of the vonical part are 1 cm and 1 cm, *i.e.*, $r = 1$ cm and $h = 1$ cm.

Then, volume of the conical part $= \frac{1}{3}\pi r^2 \times h$

$= \frac{1}{3}\pi \times (1)^2 \times (1) \text{cm}^3 = \frac{1}{3}\pi \text{cm}^3$

Therefore, the total volume of the solid $= \frac{2\pi}{3} + \frac{1}{3}\pi \text{ cm}^3 = \pi \text{ cm}^3$ [1 Mark]

18. Four tangents [1 Mark]

19. 10 cm [1 Mark]

20. Use theorem $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{\text{ar}(\Delta DEF)} = \frac{4^2}{5^2} \Rightarrow \text{ar}(\Delta DEF) = 100\text{cm}^2$ [1 Mark]

OR

By basic proportionality theorem,

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{2}{EC} \Rightarrow EC = 4$ cm. [1 Mark]

21. Let if possible $\sqrt{5} = \frac{p}{q}$, where p and q are co-prime.

$\therefore \quad 5 \times q^2 = p^2$ ....(i)

$\Rightarrow$ 5 divides $p \Rightarrow p = 5 \times p_1$; p_1 is an integer. ......(ii) [½ Mark]

From (i) and (ii), we get :

$5 \times q^2 = (5 \times p_1)^2 = 5^2 \times p_1^2 \Rightarrow q^2 = 5 \times p_1^2 \Rightarrow 5$ divides q [½ Mark]

$\Rightarrow \quad q = 5 \times q_1; q_1$ is an integer .....(iii) [½ Mark]

From (ii) and (iii), we find 5 a common factor of p and q. It contradicts that p and q are co-prime.

Hence, $\sqrt{5}$ is an irrational number. [½ Mark]

OR

First we write the prime factorisation of each of the given numbers.

$8 = 2 \times 2 \times 2 = 2^3$,

$9 = 3 \times 3 = 3^2$,

$25 = 5 \times 5 = 5^2$ [1Mark]

$\therefore$ LCM $= 2^3 \times 3^2 \times 5^2 = 8 \times 9 \times 25 = 1800$ [½ Mark]

and HCF = 1 [½ Mark]

22. $a_7 = \frac{1}{9}, a_9 = \frac{1}{7}$

$\therefore \quad a + 6d = \frac{1}{9}$ [By term formula]

$a + 8d = \frac{1}{7}$ [½ Mark]

$- \quad - \quad -$

$-2d = \frac{1}{9} - \frac{1}{7} \Rightarrow -2d = \frac{-2}{63} \Rightarrow d = \frac{1}{63}$ [½ Mark]

Now, $a + 6d = \frac{1}{9} \Rightarrow a + \frac{2}{21} = \frac{1}{9} \Rightarrow a = \frac{1}{9} - \frac{2}{21} \Rightarrow a = \frac{7-6}{63} = \frac{1}{63}$ [½ Mark]

$\therefore a_{63} = a + 62d = \frac{1}{63} + \frac{62}{63} \Rightarrow a_{63} = \frac{63}{63} = 1$ [½ Mark]

OR

Here, the last term is given. We will first have to find the number of terms.
$a = 34, d = 32 - 34 = -2, l = a_n = 10$
$\because a_n = a + (n-1)d$ [½ Mark]
$\Rightarrow 10 = 34 + (n-1)(-2) \Rightarrow (-2)(n-1) = 10 - 34$ [½ Mark]
$\Rightarrow (-2)(n-1) = -24 \Rightarrow n - 1 = 12 \Rightarrow n = 12 + 1 = 13$ [½ Mark]

Using $S_n = \frac{n}{2}(a + l)$, we have

$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 13 \times 22 = 286$ [½ Mark]

23. Total number of events = 52
In a pack of 52 playing cards, there are 2 red queens and 2 black queens, respectively. [½ Mark]
$\therefore$ Number of cards that are neither red nor queen $= 52 - (26 + 2) = 24$ [½ Mark]
Now, favourable number of events = 24

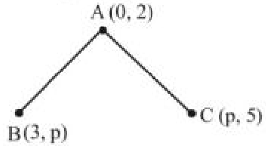
So, required probability $= \frac{24}{52} = \frac{6}{13}$ [1 Mark]

24. $(2m-1)x + 3y - 5 = 0$ ...(i)
On comparing with $a_1x + b_1y + c_1 = 0$, we get :
$a_1 = 2m - 1, b_1 = 3, c_1 = -5$
$3x + (n-1)y - 2 = 0$ ...(ii)
On comparing with $a_2x + b_2y + c_2 = 0$, we get :
$a_2 = 3, b_2 = n - 1, c_2 = -2$ [½ Mark]
For a pair of linear equations to have infinite number of solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$ [½ Mark]

$\Rightarrow 2(2m-1) = 15$ and $5(n-1) = 6 \Rightarrow m = \frac{17}{4},\ n = \frac{11}{5}$ [1 Mark]

25.



Given that : AB = AC

By distance formula : $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we get : [½ Mark]

$\sqrt{(3-0)^2+(p-2)^2}=\sqrt{(p-0)^2+(5-2)^2}$ [½ Mark]

Squaring both sides

$9+p^2+4-4p=p^2+9 \Rightarrow 4-4p=0 \Rightarrow p=1$ [½ Mark]

So, $AB=\sqrt{(3-0)^2+(1-2)^2}=\sqrt{9+1}=\sqrt{10}$ [½ Mark]

26. P (Product = 6) = P [(1, 6), (2, 3), (3, 2), (6, 1)] [1 Mark]

Probability $=\frac{4}{6^2}=\frac{4}{36}=\frac{1}{9}$ [1 Mark]

Hence, the probability that the product of the two numbers on the top of the dice is 6, will be $\frac{1}{9}$.

27. $\frac{1}{3+\sqrt{11}}=\frac{1}{3+\sqrt{11}}\times\frac{(3-\sqrt{11})}{(3-\sqrt{11})}=\frac{3-\sqrt{11}}{3^2-11}$ [1½ Marks]

$=\frac{3-\sqrt{11}}{-2}=\frac{\sqrt{11}-3}{2}=\frac{\sqrt{11}}{2}-\frac{3}{2}$ [1 Mark]

($\because$ Irrational number – Rational number = Irrational number)

Hence $\frac{1}{3+\sqrt{11}}$ is irrational. [½ Mark]

28. If α and β are the zeroes of the quadratic polynomial x^2+4x+3 then, $\alpha+\beta=-4$ and $\alpha\beta=3$

Sum of zeroes $=1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}=\frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$

$=\frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}=\frac{(\alpha+\beta)^2}{\alpha\beta}=\frac{(-4)^2}{3}=\frac{16}{3}$ [1 Mark]

Product of zeroes $=\left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)=1+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{\alpha\beta}{\alpha\beta}$

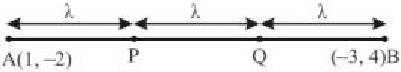
$=\frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}=\frac{(\alpha+\beta)^2}{\alpha\beta}=\frac{(-4)^2}{3}=\frac{16}{3}$ [1 Mark]

So, required polynomial

$=x^2-$(Sum of the zeroes) x + Product of the zeroes

$=x^2-\left(\frac{16}{3}\right)x+\frac{16}{3}=\frac{1}{3}(3x^2-16x+16)$ [1 Mark]

29. Let A (1, – 2) and B (– 3, 4) be the given points. Let the points of trisection be P and Q. Then, AP = PQ = QB = λ (say).



$\therefore$ PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = $\lambda : 2\lambda = 1:2$ and AQ : QB = $2\lambda : \lambda = 2:1$ [1 Mark]

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2 : 1.

Thus, the coordinates of P and Q are $P\left(\frac{1\times(-3)+2\times1}{1+2},\frac{1\times4+2\times(-2)}{1+2}\right)=P\left(\frac{-1}{3},0\right)$ [1 Mark]

$Q\left(\frac{2\times(-3)+1\times1}{2+1},\frac{2\times4+1\times(-2)}{2+1}\right)=Q\left(\frac{-5}{3},2\right)$ respectively [1 Mark]

Hence, the two points of trisection are (–1/3,0) and (–5/3,2)

OR

Here, $x_1=5, y_1=2, x_2=4, y_2=7, x_3=7$ and $y_3=-4$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)|$ [1 Mark]

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}|5(7+4)+4(-4-2)+7(2-7)|$

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}\ |(5\times11+4\times(-6)+7\times(-5)|$ [1 Mark]

$\Rightarrow$ Area of $\Delta ABC = \frac{1}{2}\ |(55-24-35)| = \frac{1}{2}|-4| = 2$ sq. units [1 Mark]

30. LHS $=\tan^2\theta+\cot^2\theta+2=(\tan^2\theta+1)+(\cot^2\theta+1)$ [1 Mark]

$=\sec^2\theta+\mathrm{cosec}^2\theta$ [$\because \tan^2\theta+1=\sec^2\theta$ and $1+\cot^2\theta=\mathrm{cosec}^2\theta$]

$=\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}$ [1 Mark]

$=\frac{1}{\sin^2\theta\cos^2\theta}=\mathrm{cosec}^2\theta\ \sec^2\theta$ [1 Mark]

= RHS (Hence Proved).

OR

$\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$

Dividing Nr. and Dr. in L.H.S. by $\cos\theta$

$\frac{1-\tan\theta}{1+\tan\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$ [1 Mark]

By comparison, we get $\tan\theta=\sqrt{3}$ [1 Mark]

$\Rightarrow\ \tan\theta=\tan60^\circ\Rightarrow\ \theta=60^\circ$ [1 Mark]

31. $ax+by=1$ …(i)

$bx+ay=\frac{2ab}{a^2+b^2}$ …(ii)

On adding (i) and (ii), we get

$(a+b)x+(a+b)y=1+\frac{2ab}{a^2+b^2}\Rightarrow(a+b)(x+y)=\frac{(a+b)^2}{a^2+b^2}$

$\Rightarrow x+y=\frac{a+b}{a^2+b^2}$ …(iii) [1 Mark]

Subtracting (ii) from (i)

$(a-b)x+(b-a)y=1-\frac{2ab}{a^2+b^2}\Rightarrow(a-b)(x-y)=\frac{(a-b)^2}{a^2+b^2}$

$\Rightarrow x - y = \frac{a-b}{a^2+b^2}$ ...(iv) [1 Mark]

Adding (iii) and (iv),

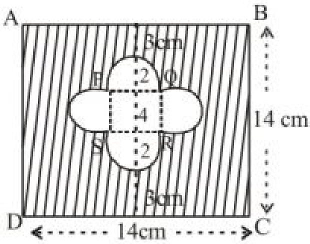
$2x = \frac{a+b}{a^2+b^2} + \frac{a-b}{a^2+b^2} = \frac{2a}{a^2+b^2} \Rightarrow x = \frac{a}{a^2+b^2}$

Subtracting (iv) from (iii)

$2y = \frac{a+b}{a^2+b^2} - \frac{a-b}{a^2+b^2} = \frac{2b}{a^2+b^2} \Rightarrow y = \frac{b}{a^2+b^2}$ [1 Mark]

Therefore, the solution is : $x = \frac{a}{a^2+b^2}$, $y = \frac{b}{a^2+b^2}$

32.



Area of sq. ABCD = $(\text{side})^2 = 196 \text{ cm}^2$ [1 Mark]

Area of small sq. = $(\text{side})^2 = 4^2 = 16 \text{ cm}^2$

Area of 4 semi-circles = $4 \times \frac{1}{2}\pi r^2 = \left[4.\frac{1}{2}(3.14)(2)^2\right]\text{cm}^2 = 25.12 \text{ cm}^2$ [1 Mark]

$\therefore$ Area of shaded region $= (196 - 16 - 25.12) \text{ cm}^2 = 154.88 \text{ cm}^2$ [1 Mark]

33. Modal class is $30 - 35$, $l = 30$, $f_1 = 25$, $f_0 = 10$, $f_2 = 7$, $h = 5$ [1 Mark]

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ [1 Mark]

$\Rightarrow$ Mode $= 30 + \left(\frac{25-10}{50-10-7}\right) \times 5 = 32.27$ approx. [1 Mark]

34. Let the required number of coins be n.

Volume of n coins = Volume of the cuboid

Then, $n \times \left\{\pi \times \left(\frac{1.75}{2}\right)^2 \times (.2)\right\} = 5.5 \times 10 \times 3.5$ [1 Mark]

$\Rightarrow n \times \frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10} = 55 \times \frac{35}{10} \Rightarrow n \times \frac{22}{7} \times \frac{7}{8} \times \frac{7}{8} \times 2 = 55 \times 35$ [1 Mark]

$\Rightarrow n = \frac{55 \times 35 \times 16}{7 \times 11}$ *i.e.*, $n = 5 \times 5 \times 16 = 400$ [1 Mark]

OR

Total surface area of solid cuboidal block = $2\,(lb + bh + hl)$ [½ Mark]

$= 2\,(15 \times 10 + 10 \times 5 + 15 \times 5) \text{ cm}^2 = 550 \text{ cm}^2$

Area of two circular bases $= 2\pi r^2 = 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 77\text{cm}^2$ [½ Mark]

Area of curved surface of cylinder $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 5 = 110\text{cm}^2$ [1 Mark]

Now, required area $= (550 + 110 - 77)\text{ cm}^2 = 583\text{ cm}^2$. [1 Mark]

35. $\sec\theta + \tan\theta = p$

since $\sec^2\theta - \tan^2\theta = 1 \Rightarrow (\sec\theta + \tan\theta) \times (\sec\theta - \tan\theta) = 1$ [½ Mark]

$\Rightarrow p \times (\sec\theta - \tan\theta) = 1 \Rightarrow \sec\theta - \tan\theta = \frac{1}{p}$ [½ Mark]

By elimination method

$\sec\theta + \tan\theta = p$ ...(i)

$\sec\theta - \tan\theta = \frac{1}{p}$ ...(ii)

\+ \+ \+

$2\sec\theta = p + \frac{1}{p} \Rightarrow 2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$ [½ Mark]

$\because \sec\theta = \frac{1}{\cos\theta} \Rightarrow \cos\theta = \frac{2p}{p^2+1}$ [½ Mark]

Subtract equation (ii) from (i)

$\sec\theta + \tan\theta = p$

$\sec\theta - \tan\theta = \frac{1}{p}$

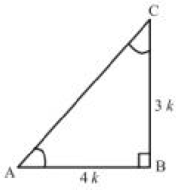
– + – [½ Mark]

$2\tan\theta = p - \frac{1}{p} \Rightarrow \tan\theta = \frac{p^2-1}{2p}$ [½ Mark]

Now, $\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \tan\theta \cdot \cos\theta = \frac{p^2-1}{2p} \times \frac{2p}{p^2+1} = \frac{p^2-1}{p^2+1}$ [1 Mark]

OR

In figure,



[½ Mark]

$3\cot A = 4$ (given) $\Rightarrow \cot A = \frac{4}{3} \Rightarrow \left(\frac{\text{Base}}{\text{Perpendicular}}\right) = \frac{AB}{BC} = \frac{4}{3}$ [½ Mark]

Let $AB = 4k$ and $BC = 3k$

In right angled ΔABC,

$AC^2 = BC^2 + AB^2$ (By pythagoras theorem)

$\Rightarrow AC = \sqrt{(4k)^2 + (3k)^2} = \pm 5k$

$\Rightarrow AC = 5k$ ($\because$ side cannot negative) [½ Mark]

Then, $\sin A = \frac{BC}{AC} = \frac{3}{5}$, $\cos A = \frac{AB}{AC} = \frac{4}{5}$ and $\tan A = \frac{BC}{AB} = \frac{3}{4}$ [½ Mark]

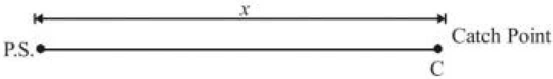
Now, $LHS = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{7}{25}$ [1 Mark]

$RHS = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$ [1 Mark]

Therefore, LHS = RHS,

$\Rightarrow \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

36.



Let the policeman catch the thief at a distance of x metres from the police station.
By the time policeman starts, the thief is at a distance of 100 m.
So, by the time thief travels ($x - 100$) more distance with a speed of 100 m/min, the policeman travels x m to catch him, increasing speed by 10 m every minute. [½ Mark]

Time taken by the thief to come to the catch point from the time the policeman starts, $t = \frac{x-100}{100}$ ... (i) [1 Mark]

Distance travelled by the policeman in first minute = 100 m, and in second minute = 110, in third minute = 120.
and in t^{th} minute = $100 + (t-1)\,10$
Total distance covered by the policeman in t minutes is

$x = \frac{t}{2}\{200 + (t-1)10\} \Rightarrow x = \frac{x-100}{2\times 100}\left\{200 + \left(\frac{x-100}{100} - 1\right)10\right\}$ [½ Mark]

$\Rightarrow 200x = (x-100)\left\{200 + \frac{(x-100-100)}{100}\times 10\right\} \Rightarrow 200\,x = (x-100)\left\{\frac{2000 + x - 200}{10}\right\}$ [½ Mark]

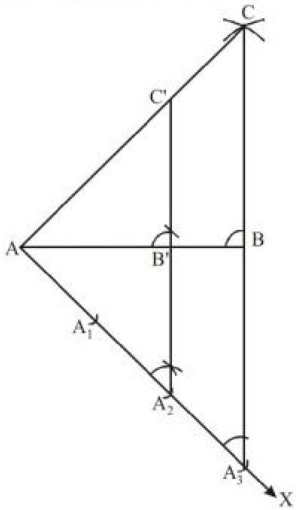
$\Rightarrow 2000x = (x-100)(x+1800) \Rightarrow 2000x = x^2 + 1800x - 100x - 180000$ [½ Mark]

$\Rightarrow x^2 - 300x - 180000 = 0 \Rightarrow (x-600)(x+300) = 0 \Rightarrow x = 600$ [½ Mark]

and $t = \frac{600-100}{100} = \frac{500}{100} = 5$ minutes

Hence, after 5 minutes, the policeman will catch the thief. [½ Mark]

37. (i) Draw a line segment AB = 5 cm.



[3 Marks]

(ii) With A as centre and radius = 7 cm, draw an arc above AB. [1 Mark]

(iii) With B as centre and radius = 6 cm, draw another arc, intersecting the arc drawn in step (ii) at C.

(iv) Join AC and BC to obtain Δ ABC.

(v) Below AB, draw a ray AX making a suitable acute angle with AB on opposite side of C with respect to AB.

(vi) Draw three arcs intersecting the ray AX at A_1, A_2, A_3 such that $AA_1 = A_1A_2 = A_2A_3$.

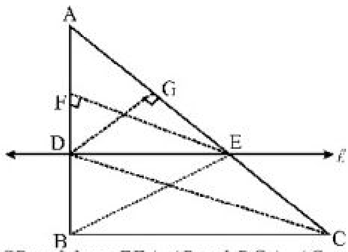
(vii) Join A_3B.

(viii) Draw $A_2B' \parallel A_3B$, meeting AB at B'.

(ix) From B', draw $B'C' \parallel BC$, meeting AC at C'.
ΔAB'C' is the required triangle, each of the whose sides is two-third of the corresponding sides of Δ ABC.

38. In ΔABC, ℓ is drawn parallel to BC which intersects AB and AC at D and E respectively.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$



Construction : Join BE and CD and draw $EF \perp AB$ and $DG \perp AC$. [½ Mark]

Proof : Δ DBE and Δ CDE are on the same base DE and between the same parallel lines DE and BC.

∴ Area (Δ BDE) = Area (Δ CDE) ....(i) [½ Mark]

Since, EF ⊥ AB therefore the height of both triangles is EF.

$$\therefore \frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta BDE)} = \frac{\frac{1}{2}\times AD\times EF}{\frac{1}{2}\times BD\times EF} = \frac{AD}{BD} \quad \text{...... (ii)}$$ [1 Mark]

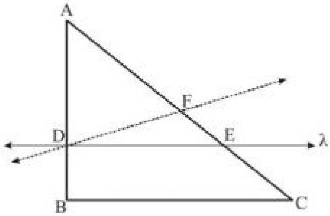
Similarly,

$$\frac{\text{Area }(\Delta ADE)}{\text{Area }(\Delta CDE)} = \frac{\frac{1}{2}\times AE\times DG}{\frac{1}{2}\times EC\times DG} = \frac{AE}{EC} \quad \text{...... (iii)}$$ [1 Mark]

Hence from (i), (ii) and (iii), we get $\frac{AD}{DB} = \frac{AE}{EC}$ [1 Mark]

OR

A triangle ABC and line λ intersecting AB at D and AC at E such that $\frac{AD}{DB} = \frac{AE}{EC}$



To prove : $DE \parallel BC$ [1 Mark]

Proof : Let us assume that DE is not parallel to BC. Then, through D there must exist some other line DF parallel to BC.

Since $DF \parallel BC$,

$\therefore$ By basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{........ (i)}$$ [1 Mark]

But, $\frac{AD}{DB} = \frac{AE}{EC}$ (given) .... (ii)

From (i) and (ii) ,we get $\frac{AF}{FC} = \frac{AE}{EC}$ [1 Mark]

On adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1 \Rightarrow \frac{AF+FC}{FC} = \frac{AE+EC}{EC} \Rightarrow \frac{AC}{FC} = \frac{AC}{EC}.\ \text{Hence, } FC = EC$$ [1 Mark]

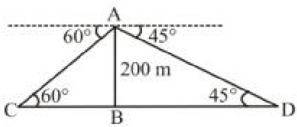
But this is impossible unless the points F and E coincide. i.e., DF coincides with DE.

Hence, $DE \parallel BC$.

39. Let AB is the height of light house = 200m

Two ships are at points C and D on either side of AB (light house)

In ΔABC,



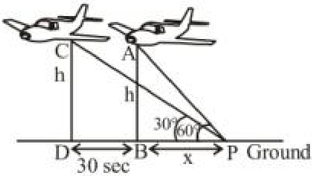
[1 Mark]

$\tan 60° = \frac{AB}{BC} \Rightarrow BC = \frac{200\sqrt{3}}{3} = \frac{200 \times 1.73}{3} = 115.33 \text{ m}$ [1 Mark]

In ΔABD, $\tan 45° = \frac{AB}{BD} \Rightarrow BD = 200$ [1 Mark]

Distance between both ships $= BC + BD = 115.33 + 200 = 315.33$ m [1 Mark]

OR



[1 Mark]

According to figure,

In ΔABP, $\tan 60° = \frac{AB}{BP} = \frac{h}{x}$ $\left(\text{Given } h = 3000\sqrt{3}\right)$

$\Rightarrow h = \sqrt{3}x \Rightarrow x = 3000 \text{ m}$ .... (i) [½ Mark]

In ΔPDC, $\tan 30° = \frac{h}{x + BD}$

$x + BD = h\sqrt{3}$ .... (ii) [½ Mark]

From (i) and (ii)

$x + BD = 3x \Rightarrow BD = 2x \Rightarrow BD = 2(3000) \Rightarrow BD = 6000 \text{ m}$ [1 Mark]

Speed of aeroplane $= \frac{BD}{30 \text{ Sec}} = \frac{6000}{30} = 200$ m /sec. [1 Mark]

40.

x_i	f_i	$x_i f_i$
3	10	30
9	p	$9p$
15	4	60
21	7	147
27	q	$27q$
33	4	132
39	1	39
Total	$\sum f_i = 26 + p + q$	$\sum x_i f_i = 408 + 9p + 27q$

[2 Marks]

Given $\sum fi = 40 \Rightarrow 26 + p + q = 40 \Rightarrow p + q = 14$ ...(i) [½ Mark]

$\therefore$ Mean, $\bar{x} = \frac{\sum x_i f_i}{\sum f_i} \Rightarrow 14.7 = \frac{408 + 9p + 27q}{40}$

$\Rightarrow 588 = 408 + 9p + 27q \Rightarrow p + 3q = 20$ ...(ii) [1 Mark]

Subtracting eq.(i) from eq. (ii),

$2q = 6 \Rightarrow q = 3$ [½ Mark]

Putting this value of q in eq. (i),

$p = 14 - q = 14 - 3 = 11$

$\therefore p = 11, q = 3$ [½ Mark]