SAMPLE PAPER 3

1. **(c)** Let radius of base of cylinder = r cm

$\therefore 346.5 = \pi r^2$ (Base area of cylinder)

$$\Rightarrow r^2 = \frac{346.5\times 7}{22} \Rightarrow r = \sqrt{\frac{346.5\times 7}{22}} \Rightarrow r = \sqrt{110.25} \Rightarrow r = 10.5 \text{ cm}$$

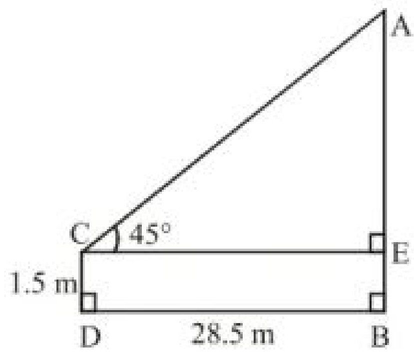
$$\therefore \text{Total surface area of cylinder} = 2\pi r(r+h) = 2\left(\frac{22}{7}\right)\times 10.5\ (10.5+24)$$

$$= 2\times 22\times 1.5\times 34.5 = 2277 \text{ sq. cm.}$$

2. **(a)** $$\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1} = \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1} = \frac{(\tan\theta+\sec\theta)[1-(\sec\theta-\tan\theta)]}{\tan\theta-\sec\theta+1}$$

$$= \frac{(\tan\theta+\sec\theta)(1-\sec\theta+\tan\theta)}{(1+\tan\theta-\sec\theta)} = \tan\theta+\sec\theta = \frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

3. **(a)** Let C be the eye of observer. And let AB be the chimney.



In $\Delta AEC : \tan 45° = \frac{AE}{CE} \Rightarrow 1 = \frac{AE}{CE} \Rightarrow AE = CE$

$AB = BE + AE = (1.5 + 28.5)\,\text{m} = 30\,\text{m}.$

4. **(d)**

Class intervals	f_i	c.f
0 – 10	5	5
10 – 20	x	5 + x
20 – 30	20	25 + x
30 – 40	15	40 + x
40 – 50	y	40 + x + y
50 – 60	5	45 + x + y
	$\Sigma f_i = 60$	

We have median = 28.5

it lies in the class interval 20-30

So, 20-30 is the median class

$l = 20$, h = 10, f = 20, F = 5 + x and $\Sigma f_i = 60$

Now, median $= l + \frac{N/2 - F}{f} \times h \Rightarrow 28.5 = 20 + \frac{30-(5+x)}{20} \times 10 \Rightarrow 8.5 = \frac{25-x}{2} \Rightarrow x = 8$

We have $\Sigma f_i = 60 \Rightarrow 45 + x + y = 60 \Rightarrow 45 + 8 + y = 60 \Rightarrow y = 7$

$\Rightarrow x + y = 8 + 7 = 15$ and $x - y = 8 - 7 = 1$

5. **(c)** DC = DE + EC = (2 + 3.5) cm = 5.5 cm

Required area = area of trap. ABCD – area of quarter circle BFEC

$= \left(\frac{1}{2}(3.5 + 5.5) \times 3.5 - \frac{1}{4} \times \frac{22}{7} \times (3.5)^2\right) cm^2 = \left(\frac{1}{2} \times 9 \times \frac{7}{2} - \frac{11}{14} \times \frac{49}{4}\right) cm^2 = \left(\frac{63}{4} - \frac{77}{8}\right) cm^2$

$= \frac{49}{8} cm^2$

6. **(a)** S = {S, W, E, E, T, C, O, R, N, S} ⇒ n(S) = 10

and E = {S, S} ⇒ n(E) = 2

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{10} = \frac{1}{5}$

7. **(a)** Let $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0 \Rightarrow 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow b = 0$

$\therefore 2a = 4 \Rightarrow a = 2$ and $b = 0$

8. **(d)** Probability of picking a blue inked pen $= \frac{\text{Number of blue inked pens}}{\text{Total number of pens}}$

$\Rightarrow \frac{2}{5} = \frac{\text{Number of blue inked pens}}{60}$

$\Rightarrow$ Number of blue inked pens $= \frac{2}{5} \times 60 = 2 \times 12 = 24$

9. **(b)** Let $f(x) = 6x^3 - 11x^2 + kx - 20$

$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0 \Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

$\Rightarrow 128 - 176 + 12k - 180 = 0 \Rightarrow 12k + 128 - 356 = 0 \Rightarrow 12k = 228 \Rightarrow k = 19$

10. **(b)** Let the annual increment be ₹ y and initial salary be ₹ x

$\therefore x + 4y = 4500$ .... (i)

and $x + 10y = 5400$ .... (ii)

Solving eqs (i) and (ii), we get

$x = 3900$ and $y = 150$

$\therefore$ Initial salary = ₹ 3900

and increment = ₹ 150

11. $\frac{61}{300} = \frac{61}{25 \times 12} = \frac{61}{5^2 \times 2^2 \times 3}$

Clearly, 300 is not of the form $2^m \times 5^n$. So the decimal expansion of $\frac{61}{300}$ is non-terminating and repeating.

Answer: Non-terminating and repeating

OR

Answer: Product

12. Since α, β are roots of $x^2 + x\sqrt{\alpha} + \beta = 0$

$\Rightarrow \quad \alpha^2 + \alpha\sqrt{\alpha} + \beta = 0$ ...(i)

and $\beta^2 + \beta\sqrt{\alpha} + \beta = 0$ ...(ii)

Multiply equation (i) by β and equation (ii) by α and subtract

$\alpha^2\beta + \alpha\beta\sqrt{\alpha} + \beta^2 = 0$

$\alpha\beta^2 + \alpha\beta\sqrt{\alpha} + \alpha\beta = 0$

(–) (–) (–)

$\alpha\beta(\alpha - \beta) + \beta(\beta - \alpha) = 0$

$\Rightarrow \quad (\alpha\beta - \beta)(\alpha - \beta) = 0$

$\Rightarrow \quad \alpha\beta - \beta = 0$ $(\because \alpha - \beta = 0 \Rightarrow \alpha = \beta$ which is not possible)

$\Rightarrow \quad (\alpha - 1)\beta = 0 \Rightarrow \alpha - 1 = 0 \Rightarrow \alpha = 1$

Put $\alpha = 1$ in (i), we get : $\beta = -2$

Answer: $\alpha = 1, \beta = -2$

13. $\frac{a+b+c}{3} = 0 \Rightarrow a+b+c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$ (Using identity).

Answer: 3 abc

14. Let the heights of two cones be h and 3h, and their radii be 3r and r respectively.

$$\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{\frac{1}{3}\pi(3r)^2 h}{\frac{1}{3}\pi r^2 \times 3h} = \frac{3}{1}$$

Answer: 3:1

15. $t_8 = a + 7d$, $t_{12} = a + 11d$

According to question, $8t_8 = 12t_{12}$ (given)

$\Rightarrow 8(a + 7d) = 12(a + 11d)$

$\Rightarrow 8a + 56d = 12a + 132d$

$\Rightarrow 8a - 12a + 56d - 132d = 0$

$\Rightarrow -4a - 76d = 0 \quad \Rightarrow a + 19d = 0$

Now, $t_{20} = a + 19d = 0$

Answer: 0

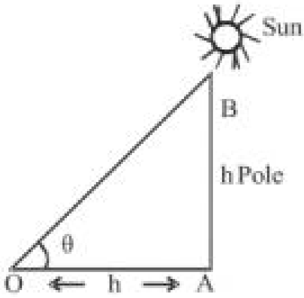
16. $\angle BAT = \angle BTP$ ..... [Alternate segment]

$\angle ABT + \angle BAT + \angle BTA = 180°$

$\angle ABT = 60°$ [1 Mark]

17. $2 + \sqrt{3}$ [1 Mark]

18. Let AB be the pole of height h.
OA be the shadow of the pole.
In rt. ΔOAB,



$$\frac{AB}{OA} = \tan\theta$$

$$\Rightarrow \frac{h}{h} = \tan\theta \;(\because OA = AB = h)$$

$\Rightarrow \tan\theta = 1 \Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$ [1 Mark]

19. Yes, radius of the circle is breadth of the rectangle. [1 Mark]

20. N = 10 (even).

$$\text{Median} = \frac{1}{2}\,(5^{th}\text{ obs.} + 6^{th}\text{ obs.})$$

$$= \frac{1}{2}\,(2x - 8 + 2x + 10) = 2x + 1$$

$2x + 1 = 25 \Rightarrow x = 12.$ [1 Mark]

OR

Variate are 3, 4, 6, 7, 8, 14

$$\therefore \text{ Mean, } (\bar{x}) = \frac{3+4+6+7+8+14}{6} = \frac{42}{6} = 7$$

Sum of deviations from $\bar{x} = 7$ is

$(3-7)+(4-7)+(6-7)+(7-7)+(8-7)+(14-7)$

$= -4-3-1+0+1+7 = 0$ [1 Mark]

21. The Prime factorisation of 66 & 486 gives

$66 = 2 \times 3 \times 11$

$486 = 2 \times 3 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^5$ [½ Mark]

$\therefore$ The LCM of these two integer is : $2 \times 3^5 \times 11 = 5346$ [½ Mark]

$$\text{HCF}(66, 486) = \frac{66 \times 486}{\text{LCM}(66, 486)} = \frac{66 \times 486}{5346} = 6$$ [1 Mark]

22. Given : $a_5 + a_9 = 30$

$a_{25} = 3a_8$

Now, $a + 4d + a + 8d = 30 \Rightarrow 2a + 12d = 30$

$\Rightarrow \quad a + 6d = 15$ ...(i) [½ Mark]

and, $a + 24d = 3a + 21d \Rightarrow 2a - 3d = 0$ ...(ii) [½ Mark]

From eqs. (i) and (ii)

$$\begin{array}{r} 2a + 12d = 30 \\ \underset{-}{}2a \underset{+}{-} 3d = \underset{-}{}0 \\ \hline 15d = 30 \end{array} \quad \Rightarrow d = 2$$ [½ Mark]

Now, put d = 2 in eq. (i) : $a + 12 = 15 \Rightarrow a = 3$

Required A.P. = 3, 5, 7, ... [½ Mark]

23. Let the number of blue balls = x

$\therefore$ Total number of balls = 5 + x

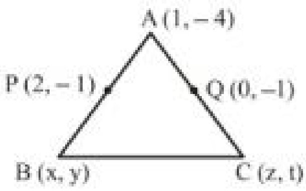
P (blue ball) = $\dfrac{x}{5+x}$ [½ Mark]

P (red ball) = $\dfrac{5}{5+x}$ [½ Mark]

Given that P (blue) = 2 × P (red) $\Rightarrow \dfrac{x}{5+x} = 2 \times \dfrac{5}{5+x} \Rightarrow \dfrac{x}{5+x} = \dfrac{10}{5+x}$

On solving we get x = 10 [1 Mark]

24.



P is the mid-point of AB

$\therefore$ $x + 1 = 4 \Rightarrow x = 3$ [By Mid-Point formula]

$y - 4 = -2 \Rightarrow y = 2 \Rightarrow B(3, 2)$ [½ Mark]

Similarly, $z + 1 = 0 \Rightarrow z = -1$ and $t - 4 = -2 \Rightarrow t = 2$

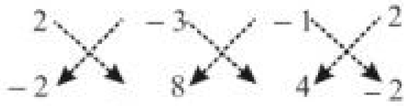
$\Rightarrow$ C(–1, 2) [½ Mark]

$\therefore$ Area $\Delta ABC = \dfrac{1}{2}\left[1(2-2)+3(2+4)-1(-4-2)\right] \Rightarrow \dfrac{1}{2} \times 24 = 12$ sq units [1 Mark]

OR

Let Δ be the area of the triangle formed by the given points

We have,



[½ Mark]

$\therefore$ $\Delta = \dfrac{1}{2}\left|\{2 \times 8 + (-3) \times 4 + (-1) \times (-2)\} - \{(-3) \times (-2) + (-1) \times 8 + 2 \times 4\}\right|$ [½ Mark]

$\Rightarrow$ $\Delta = \dfrac{1}{2}\left|(16 - 12 + 2) - (6 - 8 + 8)\right|$ [½ Mark]

$\Rightarrow$ $\Delta = \dfrac{1}{2}\left|6 - 6\right| = 0$

Hence, given points are collinear. [½ Mark]

25. Since, (3, a) lies on the line $2x - 3y = 5$

So, $2 \times 3 - 3a = 5$ [1 Mark]

$\Rightarrow$ $6 - 3a = 5 \Rightarrow a = \dfrac{1}{3}$ [1 Mark]

OR

$3x + 2y - 7 = 0$ ... (i)

$4x + y - 6 = 0$ ... (ii)

From (ii), $y = 6-4x$ [½ Mark]
Value of y put in eqn. (i)
$3x+2y-7=0$
$\Rightarrow\quad 3x+2(6-4x)-7=0$
$\Rightarrow\quad 3x+12-8x-7=0$
$\Rightarrow\quad 5x=5$
$\therefore\quad x=1$ [½ Mark]
Substitute the value of x in eq(ii) to get value of y,
$4x+y-6=0$ [½ Mark]
$\Rightarrow\quad 4(1)+y-6=0$
$\Rightarrow\quad 4+y-6=0$
$\Rightarrow\quad y-2=0$
$\therefore\quad y=2$
Hence, values of x and y are 1 and 2. [½ Mark]

26. Total number of possible outcomes when two dice are thrown simultaneously = 36 [½ Mark]
Sum of the numbers appearing on the dice is a prime number i.e., 2, 3, 5, 7 and 11
So, the possible outcomes are (1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6) and (6, 5).
Number of possible outcomes = 15 [½ Mark]

$\therefore$ required probability $=\frac{15}{36}=\frac{5}{12}$ [1 Mark]

27. Given integers are 81 and 237 such that $81 < 237$.
Applying *Euclid's division lemma* to 81 and 237, we get

$237 = 81\times 2+75$ ...(i) $\left[\because 81\overline{)237}(2;\ 162;\ 75\right]$ [½ Mark]

Since the remainder $75\neq 0$. So, consider the divisor 81 and the remainder 75 and apply division lemma to get

$81 = 75\times 1+6$ ...(ii) $\left[\because 75\overline{)81}(1;\ 75;\ 6\right]$ [½ Mark]

We consider the new divisor 75 and the new remainder 6 and apply division lemma to get

$75 = 6\times 12+3$ ...(iii) $\left[\because 6\overline{)75}(12;\ 72;\ 3\right]$ [½ Mark]

We consider the new divisor 6 and the new remainder 3 and apply division lemma to get

$6 = 3\times 2+0$ ...(iv) $\left[\because 3\overline{)6}(2;\ 6;\ 0\right]$ [½ Mark]

The remainder at this stage is zero. So, the last divisor i.e. 3 is the HCF of 81 and 237.
To represent the HCF as a linear combination of the given two numbers, we start from the last but one step and successively eliminate the previous remainders as follows :
From (iii), we have

$3 = 75-6\times 12 \Rightarrow 3 = 75-(81-75\times 1)\times 12$ [Substituting $6 = 81-75\times 1$ obtained from (ii)]

$\Rightarrow\quad 3 = 75-12\times 81+12\times 75 \Rightarrow 3 = 13\times 75-12\times 81$
$\Rightarrow\quad 3 = 13\times(237-81\times 2)-12\times 81$ [Substituting $75 = 237-81\times 2$ obtained from (i)] [½Mark]
$\Rightarrow\quad 3 = 13\times 237-26\times 81-12\times 81 \Rightarrow 3 = 13\times 237-26\times 81-12\times 81 \Rightarrow 3 = 13\times 237-38\times 81$
$\Rightarrow\quad 3 = 237x+81y$, where $x = 13$ and $y = -38$. [½ Mark]

28. Perimeter of shaded region = Perimeter (QTR + QAP + PSR) [1 Mark]

$= \pi\left[5+\frac{3}{2}+\frac{7}{2}\right] = \pi\left[\frac{20}{2}\right] = 10\pi = 31.4$ cm [2 Marks]

29. Let the radius of spherical marble = 0.7 cm [½ Mark]

Volume of 1 marble $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.7)^3\ cm^3$ [½ Mark]

Volume of 150 marble $= 200\pi(0.7)^3\ cm^3$ [½ Mark]

Let h be the rise in the height of water

∴ Volume of water raised = Volume of 150 marbles [½ Mark]

So, $\pi \times 7^2 \times h = 200\pi(0.7)^3 \Rightarrow h = \frac{200\times7\times7\times7}{7\times7\times10\times10\times10} \Rightarrow h = 1.4$ cm [1 Mark]

OR

Let the radius of hemisphere = r

Now, volume of hemisphere $= \frac{2}{3}\pi r^3$ [1 Mark]

Surface area of hemisphere $= 3\pi r^2$ [1 Mark]

A.T.Q, volume of hemisphere = surface area of hemisphere

$\Rightarrow \frac{2}{3}\pi r^3 = 3\pi r^2 \Rightarrow r = \frac{9}{2}$ units [1 Mark]

So, the required diameter of hemisphere = 2r = 9 units.

30. Suppose the co-ordinates of vertex D are (x, y), then

Mid-point of AC = Mid-point of BD (For parallelogram ABCD) [½ Mark]

$\left(\frac{3-6}{2}, \frac{-4+2}{2}\right) = \left(\frac{-1+x}{2}, \frac{-3+y}{2}\right)$ [½ Mark]

$\left(\frac{-3}{2}, -1\right) = \left(\frac{x-1}{2}, \frac{y-3}{2}\right) \Rightarrow x = -2, y = 1$ [½ Mark]

Area of $\Delta ABC = \left|\frac{1}{2}[3(-3-2)-1(2+4)-6(-4+3)]\right| = 7.5$ sq. units [1 Mark]

Similarly, area of $\Delta ACD = 7.5$ sq. units [By property of parallelogram]

Hence, area of ABCD = 7.5 + 7.5 = 15 sq.units. [½ Mark]

31.

Class	f	$c.f.$
0 – 10	5	5
10 – 20	x	$x+5$
20 – 30	20	$x+25$
30 – 40	15	$x+40$
40 – 50	y	$x+y+40$
50 – 60	5	$x+y+45$
	$\sum f = 60$	

[1 Mark]

From table, $N = 60 = x + y + 45 \Rightarrow x + y = 60 - 45 = 15$ ... (i) [½ Mark]

Since, Median = 28.5

Median class = 20 – 30

Median = $l + \dfrac{\left(\dfrac{N}{2} - c.f.\right)}{f} \times h$ [½ Mark]

$\Rightarrow 28.5 = 20 + \dfrac{[30-(x+5)]}{20} \times 10 \Rightarrow 8.5 = \dfrac{25-x}{2} \Rightarrow 25 - x = 17 \Rightarrow x = 25 - 17 = 8$ [½ Mark]

From (i), $y = 15 - 8 = 7$ [½ Mark]

OR

Here, n = 100, $\overline{x} = 40$

We know, $\overline{x} = \dfrac{1}{n}\left(\sum x_i\right) \Rightarrow 40 = \dfrac{1}{100}\left(\sum x_i\right)$ [1 Mark]

$\therefore$ Incorrect value of $\sum x_i = 4000$

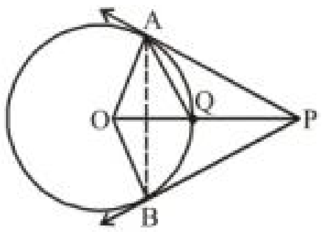
Now, correct value of $\sum x_i = 4000 - 83 + 53 = 3970$ [1 Mark]

$\therefore$ Correct mean $= \dfrac{\text{correct value of } \sum x_i}{n} = \dfrac{3970}{100} = 39.7$ [1 Mark]

So, the correct mean is 39.7

32. Let OP meet the circle at Q. Join AQ. As OP is equal to the diameter of the circle and OQ is radius, so OQ = QP i.e. Q is mid-point of OP. Since PA is tangent to the circle at A and OA is its radius, OA ⊥ AP i.e. ∠OAP = 90°.

In right triangle OAP, Q is mid-point of hypotenuse,



$\therefore$ AQ = OQ = QP

Also OA = OQ (radii of same circle)

$\Rightarrow$ OA = OQ = AQ $\Rightarrow$ ΔOAQ is equilateral

$\Rightarrow$ ∠AOQ = 60° $\Rightarrow$ ∠AOP = 60°. [1 Mark]

In ΔOAP, ∠OPA + ∠AOP + ∠OAP = 180°

$\Rightarrow$ ∠OPA + 60° + 90° = 180° $\Rightarrow$ ∠OPA = 30°

$\Rightarrow$ ∠APB = 60° ($\therefore$ OP is bisector of ∠APB) [½ Mark]

Also PA = PB $\Rightarrow$ ∠PAB = ∠PBA.

In ΔPAB, ∠PAB + ∠PBA + ∠APB = 180° [½ Mark]

$\Rightarrow$ 2 ∠PAB + 60° = 180° $\Rightarrow$ ∠PAB = 60°

$\Rightarrow$ Triangle ABP is equilateral. [1 Mark]

33. $\therefore$ Let $x^2 = u, y^2 = v \Rightarrow 2u + 3v = 35$ and $\frac{u}{2} + \frac{v}{3} = 5$ [½ Mark]

$\Rightarrow \quad 2u + 3v = 35$ ...(i)

$\Rightarrow \quad 3u + 2v = 30$ ...(ii) [½ Mark]

Multiply (i) by 3 and (ii) by 2 and subtracting (ii) from (i), we have

$\Rightarrow \quad 6u - 6u + 9v - 4v = 105 - 60 \Rightarrow 5v = 45 \Rightarrow v = 9$ [1 Mark]

Substituting $v = 9$ in (i), we get

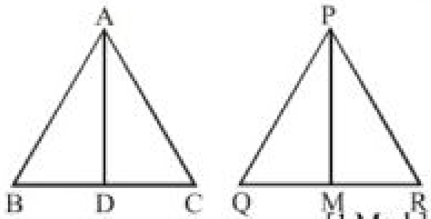
$2u + 27 = 35 \Rightarrow 2u = 8 \Rightarrow u = 4 \Rightarrow x^2 = 4, y^2 = 9$

$\therefore \; x = \pm 2, y = \pm 3$ is the required solution. [1 Mark]

34. **Given :** ΔABC and ΔPQR, in which,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To Prove : $\Delta ABC \sim \Delta PQR$ [1 Mark]



Proof : $\frac{AB}{PQ} = \frac{BC}{QR}$ (Given)

$$\frac{2BD}{2QM} = \frac{BC}{QR} \therefore \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$\therefore \quad \Delta ABD \sim \Delta PQM$ (By SSS) [1 Mark]

$\therefore \quad \angle B = \angle Q$ (By corresponding angles of similar triangles)

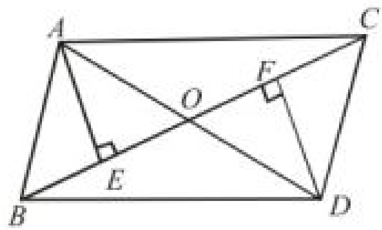
Now, In ΔABC and ΔPQR, $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$

$\therefore$ **$\Delta ABC \sim \Delta PQR$ (By SAS)** [1 Mark]

OR

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction : Draw $AE \perp BC$ and $DF \perp BC$.



[½ Mark]

Proof :

In ΔAOE and ΔDOF,

$\angle AOE = \angle DOF$ (Vertically opposite angles)

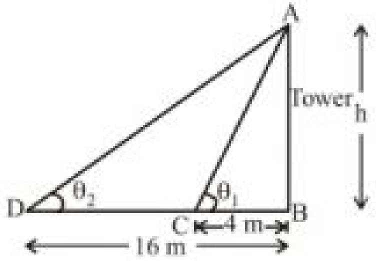
$\angle AEO = \angle DFO = 90°$ (Construction)

$\Rightarrow \quad \Delta AOE \sim \Delta DOF$ (By AA Similarity) [½ Mark]

$\therefore \quad \frac{AO}{DO} = \frac{AE}{DF}$ ...(i) [1 Mark]

Now, $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF} = \frac{AO}{DO}$ [1 Mark]

35. Suppose AB be a tower and there are two points C and D at the distances of 4 m and 16 m from the foot of the tower respectively.



[1 Mark]

Since, the angles of elevation from C and D of the top of the tower are complementary.

So, $\theta_1 + \theta_2 = 90°$ ... (i) [½ Mark]

Let the height of the tower be h.

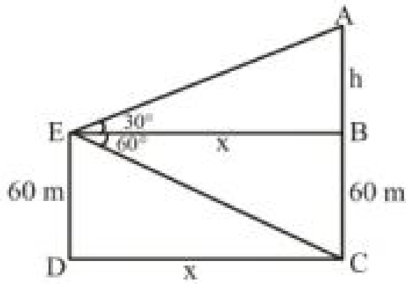
Then, from equation (i), $\tan(\theta_1 + \theta_2) = \tan 90°$ [½ Mark]

$\Rightarrow \dfrac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = \dfrac{1}{0} \Rightarrow 1 - \tan\theta_1 \tan\theta_2 = 0 \Rightarrow \tan\theta_1 \tan\theta_2 = 1$ [1 Mark]

$\Rightarrow \dfrac{h}{4} \times \dfrac{h}{16} = 1 \Rightarrow h^2 = 64 \Rightarrow h = 8$ m ($\because$ height cannot be negative) [1 Mark]

Hence, the height of the tower is 8 m.

OR



[1 Mark]

Let ED is 60 m high building and AC is the tower of height = (h + 60) m. Let distance b/w tower and building is x m. [½ Mark]

Now, In ΔABE

$\tan 30° = \dfrac{AB}{BE} = \dfrac{h}{x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x} \Rightarrow x = h\sqrt{3}$ ... (i) [½ Mark]

In ΔBCE, $\tan 60° = \dfrac{BC}{BE} \Rightarrow \sqrt{3} = \dfrac{60}{x} \Rightarrow x = \dfrac{60}{\sqrt{3}} = 20\sqrt{3}$ m [1 Mark]

Now, $h = \dfrac{x}{\sqrt{3}} = 20$ m [½ Mark]

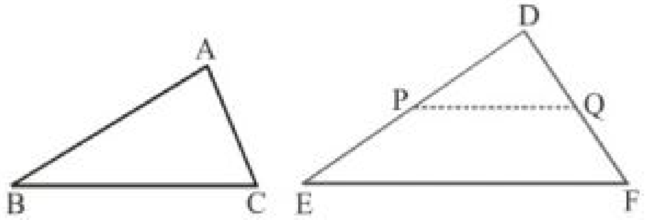
Now, distance between tower and building is $x = 20\sqrt{3}$ m

Difference between the heights of tower and building is 20 m. [½ Mark]

36. **Given :** Two ΔABC and ΔDEF, such that $\angle A = \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on side DF such that $AB = DP$ and $AC = DQ$, join PQ. [1 Mark]
Proof : In ΔABC and ΔDPQ



$AB = DP$ (By Construction)

$AC = DQ$ (By Construction)

$\angle A = \angle D$ (Given)

By SAS rule of congruence

$\Delta ABC \cong \Delta DPQ$ .... (i) [1 Mark]

$\frac{AB}{DP} = \frac{AC}{DQ}$ (By Construction) ... (ii)

and $\frac{AB}{DE} = \frac{AC}{DF}$ (Given) ... (iii)

From (ii) and (iii), $\frac{DP}{DE} = \frac{DQ}{DF}$ [1 Mark]

By converse of basic Proportionality theorem $PQ \parallel EF$

So, $\angle DPQ = \angle E$ and $\angle DQP = \angle F$ (corresponding angles) [½ Mark]

Consequently, by AA similarity, $\Delta DPQ \sim \Delta DEF$ ... (iv)

From (i) and (iv), we get, $\Delta ABC \sim \Delta DPQ \sim \Delta DEF$

$\Rightarrow$ $\Delta ABC \sim \Delta DEF$ [½ Mark]

OR

Consider AB as a vertical pole and BC its shadow. And also consider (PM = h) as a tower and MN its shadow, as shown figure.

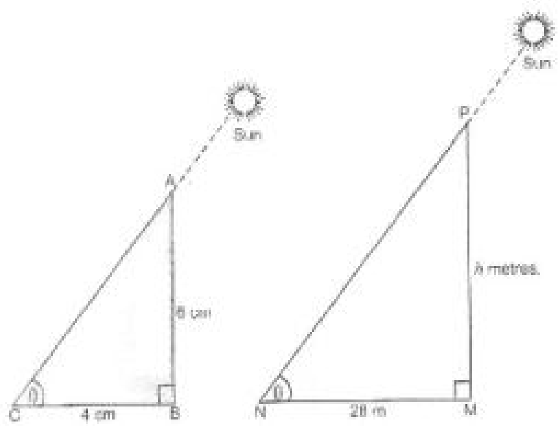
Let θ be the altitude of sun.

$\therefore \angle C = \angle N$ (Each = θ)

$\angle B = \angle M$ (Each = 90°) [½ Mark]

$\therefore$ by AA similarity criterion :

$\Delta ABC \sim \Delta PMN$. [1 Mark]



[1 Mark]

$\Rightarrow \frac{AB}{PM} = \frac{BC}{MN} \Rightarrow \frac{AB}{BC} = \frac{PM}{MN}$ [½ Mark]

$\Rightarrow \frac{6}{4} = \frac{h}{28} \Rightarrow h = \frac{6 \times 28}{4} = 42\text{m}.$ [1 Mark]

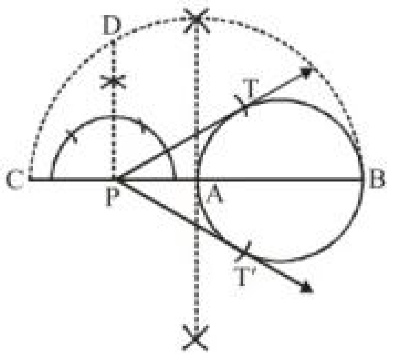
37. **LHS** $= \frac{\tan^2\theta}{\tan^2\theta - 1} + \frac{\text{cosec}^2\theta}{\sec^2\theta - \text{cosec}^2\theta} = \frac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta} - 1} + \frac{\frac{1}{\sin^2\theta}}{\frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta}}$ [1 Mark]

$= \frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta\cos^2\theta}{\sin^2\theta - \cos^2\theta}$ [1 Mark]

$= \frac{\sin^2\theta}{\sin^2\theta - \cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta - \cos^2\theta}$ [1 Mark]

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta} = \frac{1}{\sin^2\theta - \cos^2\theta}$ **= RHS** [1 Mark]

38. **Steps of construction :**



[3 Marks]

(i) Draw a circle of radius 4 cm.
(ii) Take a point P outside the circle and draw a secant PAB, intersecting the circle at A and B.
(iii) Produce AP to C such that $AP = CP$.
(iv) Draw a semi-circle with CB as diameter.
(v) Draw $PD \perp CB$, intersecting the semi-circle at D.
(vi) With P as centre and PD as radius draw arcs to intersect the given circle at T and T'.
(vii) Join PT and PT'. Then, PT and PT' are the required tangents. [1 Mark]

39. The equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For coincident (Repeated roots) D = 0 [½ Mark]

$\Rightarrow (2mc)^2 - 4(1 + m^2)(c^2 - a^2) = 0$ [½ Mark]

$\Rightarrow 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2) = 0$ [½ Mark]

$\Rightarrow m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$ [½ Mark]

$\Rightarrow m^2a^2 - c^2 + a^2 = 0$ [½ Mark]

$\Rightarrow m^2a^2 + a^2 = c^2 \Rightarrow a^2(1 + m^2) = c^2$ [1 Mark]

$\Rightarrow c = \pm a\sqrt{1 + m^2}$. **Hence proved.** [½ Mark]

40. Let $(a-3d)$, $(a-d)$, $(a+d)$, $(a+3d)$ are the four numbers in A.P. [1 Mark]
A.T.Q.

$\Rightarrow (a-3d)+(a-d)+(a+d)+(a+3d)=50 \Rightarrow a=\frac{25}{2}$ [1 Mark]

also, $(a+3d)=4(a-3d) \Rightarrow 5d=a$ [½ Mark]

$\Rightarrow d=\frac{5}{2}$ [½ Mark]

$\therefore$ the numbers are $\left(\frac{25}{2}-\frac{15}{2}\right),\left(\frac{25}{2}-\frac{5}{2}\right),\left(\frac{25}{2}+\frac{5}{2}\right)$ & $\left(\frac{25}{2}+\frac{15}{2}\right)$ or 5, 10, 15 and 20. [1 Mark]

OR

$a_3+a_7=6$ (Given)

$\Rightarrow (a+2d)+(a+6d)=6$ $\{\because a_n=a+(n-1)d\}$

$\Rightarrow 2a+8d=6$

$\Rightarrow a+4d=3$ .....(1) [½ Mark]

Also, product = 8 (Given)

i.e. $a_3\times a_7=8 \Rightarrow (a+2d)(a+6d)=8 \Rightarrow (a+4d-2d)(a+4d+2d)=8$ [½ Mark]

$\Rightarrow (3-2d)(3+2d)=8$ [from (1)]

$\Rightarrow 9-4d^2=8 \quad \Rightarrow d=\pm\frac{1}{2}$ $\{\because (a+b)(a-b)=(a^2-b^2)\}$ [1 Mark]

Taking $d=\frac{1}{2}$, then a = 1 [from (1)]

$\therefore S_n=\frac{n}{2}[2a+(n-1)d]$

$S_{16}=\frac{n}{2}\left[2(1)+(16-1)\times\frac{1}{2}\right]=8\left[2+\frac{15}{2}\right]=8\times\frac{19}{2}=76$ [1 Mark]

Taking $d=-\frac{1}{2}$, then $a=5$ [From (1)]

$S_{16}=\frac{16}{2}\left[2\times(5)+(16-1)\left(\frac{-1}{2}\right)\right]=8\left[10-\frac{15}{2}\right]=8\left[\frac{20-15}{2}\right]=8\times\frac{5}{2}=20$

$\therefore S_{16}=20, 76$ [1 Mark]