SAMPLE PAPER 4

1. (c) 1st wheel makes 1 revolutions per sec

2nd wheel makes $\frac{6}{10}$ revolutions per sec

3rd wheel makes $\frac{4}{10}$ revolutions per sec

In other words 1st, 2nd and 3rd wheel take 1, $\frac{5}{3}$ and $\frac{5}{2}$ seconds respectively to complete one revolution.

L.C.M of 1, $\frac{5}{3}$ and $\frac{5}{2} = \frac{\text{L.C.M of } 1, 5, 5}{\text{H.C.F of } 1, 3, 2} = 5$

Hence, after every 5 seconds the red spots on all the three wheels touch the ground.

2. (c) Let α and β be two roots of equation, $\beta < \alpha$, so $\alpha - \beta = 1 \Rightarrow \alpha = 1 + \beta$

Sum of roots : $\alpha + \beta = \frac{b}{1} \Rightarrow 1 + \beta + \beta = b \Rightarrow 2\beta = b - 1 \Rightarrow \beta = \frac{b-1}{2}$

Product of roots : $\alpha\beta = \frac{c}{1} \Rightarrow (1+\beta)\beta = c \Rightarrow \beta + \beta^2 = c \Rightarrow \frac{b-1}{2} + \left(\frac{b-1}{2}\right)^2 = c$

$\Rightarrow \frac{b-1}{2} + \frac{b^2 - 2b + 1}{4} = c \Rightarrow 2b - 2 + b^2 - 2b + 1 = 4c \Rightarrow b^2 - 4c - 1 = 0$

3. (a) Only I & III are correct.

4. (b)

C.I.	x_i	f_i	$f_i x_i$
0 - 10	5	8	40
10 - 20	15	12	180
20 - 30	25	10	250
30 - 40	35	11	385
40 - 50	45	9	405
		50	1260

We have $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1260}{50} = 25.2$

5. (d) 28, 22, x, y, 4 are in AP

$\Rightarrow \quad 22 - 28 = x - 22 = y - x = 4 - y \Rightarrow -6 = x - 22$ and $-6 = 4 - y$

$\Rightarrow \quad x = -6 + 22$ and $y = 4 + 6 \Rightarrow x = 16$ and $y = 10$

6. (a) $\Delta FBE \sim \Delta FDA \quad [\because \angle 1 = \angle 2, \angle 4 = \angle 3]$

$\frac{EF}{FA} = \frac{FB}{DF}$

7. (a) The two vertices of the triangle are given as A(6, 4) & B(–2, 2). Let the third vertex be C (a, b). Then the coordinates of its centroid is

$G\left(\frac{6-2+a}{3}, \frac{4+2+b}{3}\right) = G\left(\frac{4+a}{3}, \frac{6+b}{3}\right)$

But it is given that centroid is G(3, 4) $\therefore \frac{4+a}{3} = 3 \ \& \ \frac{6+b}{3} = 4$

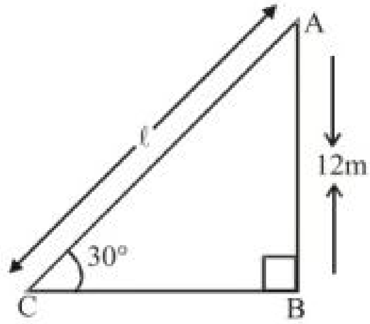
$\Rightarrow \quad a = 5 \ \& \ b = 6$

8. **(d)** Let AB be the vertical pole.

Let AC be the rope tied at point C on the ground.

Let length of rope = ℓ m.

In ΔABC, $\text{cosec } 30° = \frac{\ell}{12} \Rightarrow 2 = \frac{\ell}{12} \Rightarrow \ell = 24$ m



9. **(a)** $\frac{1+\cos A}{\sin A} + \frac{\sin A}{1+\cos A} = \frac{(1+\cos A)^2 + \sin^2 A}{\sin A(1+\cos A)} = \frac{1+2\cos A + \cos^2 A + \sin^2 A}{\sin A(1+\cos A)} = \frac{1+2\cos A+1}{\sin A(1+\cos A)}$

$= \frac{2+2\cos A}{\sin A(1+\cos A)} = \frac{2(1+\cos A)}{\sin A(1+\cos A)} = \frac{2}{\sin A} = 2\text{cosec } A$

10. **(b)** Number of balls = $\frac{\text{volume of lead}}{\text{volume of a ball}} = \frac{(44)^3}{4/3\pi r^3}$ [$\because$ volume of a cube = $(\text{edge})^3$]

$= \frac{44 \times 44 \times 44}{\frac{4}{3} \times \frac{22}{7} \times (2)^3}$ [$\because$ Diameter of a ball = 4 cm. $\therefore$ radius of a ball = 2 cm.]

$= \frac{44 \times 44 \times 44 \times 21}{4 \times 22 \times 8} = 2541$

11. Let the third side be x cm. Then, by Pythagoras theorem, we have
$p^2 = q^2 + x^2$

$\Rightarrow x^2 = p^2 - q^2 = (p-q)(p+q) = p+q$ [$\because p - q = 1$]

$\Rightarrow x = \sqrt{p+q} = \sqrt{2q+1}$ [$\because p - q = 1 \therefore p = q + 1$]

Hence, the length of the third side is $\sqrt{2q+1}$ cm.

Answer : $\sqrt{2q+1}$

12. Volume of cylinder = $\pi r^2 h$

$\Rightarrow 448\pi = \pi r^2 \times 7 \Rightarrow r^2 = \frac{448}{7} \Rightarrow r = \sqrt{\frac{448}{7}} \Rightarrow r = 8$ cm.

$\therefore$ L. S. A or C. S. A = $2\pi rh = 2 \times \frac{22}{7} \times 8 \times 7 = 352 \text{ cm}^2$

Answer : 352 cm^2

13. Area of the shaded region $= \frac{40°}{360°} \times \frac{22}{7} \times (7)^2 - \frac{40°}{360°} \times \frac{22}{7} \times (3.5)^2$

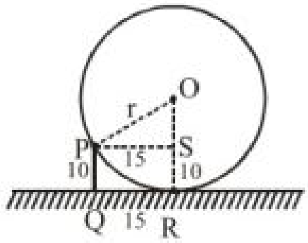
$= \frac{1}{9} \times \frac{22}{7} \times (7^2 - 3.5^2) = \frac{1}{9} \times \frac{22}{7} \times \left(49 - \frac{49}{4}\right) = \frac{1}{9} \times \frac{22}{7} \times \frac{49}{4} \times 3 = \frac{77}{6} \text{cm}^2$

Answer : $\frac{77}{6}$ cm^2

14. Total number of sample points = 20
Total number of favourable outcomes = 3
Required probability = $\frac{3}{20}$

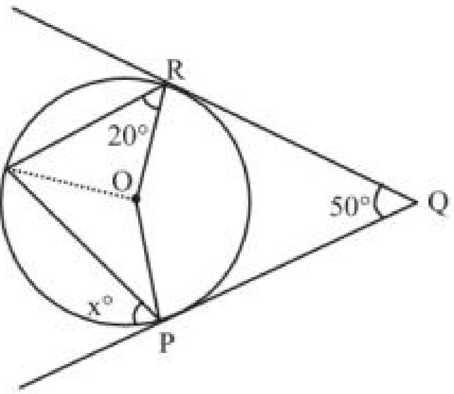
Answer : $\frac{3}{20}$

15. In right Δ OSP,



$OP^2 = PS^2 + OS^2$
$\Rightarrow r^2 = 225 + (r-10)^2 \Rightarrow r^2 = 225 + r^2 - 20r + 100 \Rightarrow 20r = 325$
$\Rightarrow 2r = 32.5$
Hence, diameter = 32.5 cm.
Answer : 32.5 cm

OR

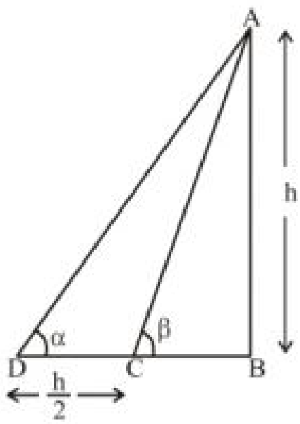


$\angle POR + \angle PQR = 180°$
$\therefore \angle POR = 180° - 50° = 130°$

$\angle PSR = \frac{1}{2} \angle POR$

$\therefore \angle PSR = \frac{1}{2} \times 130° = 65°$
$\Rightarrow \angle PSR = \angle OSP + \angle OSR$
$\Rightarrow \angle PSR = \angle OSP + 20°$ [$\because \angle OSP = \angle OSR$]
$\Rightarrow 65° = \angle OSP = 20°$
$\Rightarrow \angle OSP = 45°$
$\Rightarrow \angle OPX = 90°$ [$\because$ PX is a tangent]
$\Rightarrow \angle SPX + \angle OPS = 90°$
$x° + \angle OSP = 90°$ [$\because \angle OPS = \angle OSP$]
$x° = 90° - 45° = 45°$
Answer : 45°

16. Here, In ΔABD,



$$\cot\alpha = \frac{BD}{AB} = \frac{\frac{h}{2} + BC}{h} \quad \text{....(i)}$$

also, In ΔABC,

$$\cot\beta = \frac{BC}{AB} = \frac{BC}{h}$$

Now, $\cot\alpha - \cot\beta = \frac{\frac{h}{2} + BC}{h} - \frac{BC}{h} = \frac{h + 2BC}{2h} - \frac{BC}{h} = \frac{h + 2BC - 2BC}{2h} = \frac{1}{2}$ [1 Mark]

17. 7 : 4 [1 Mark]

18. $\sec^2\theta + \tan^2\theta = \frac{5}{3}$ or $1 + \tan^2\theta + \tan^2\theta = \frac{5}{3}$

$2\tan^2\theta = \frac{2}{3}$ or $\tan^2\theta = \frac{1}{3}$ or $\tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan 30^o$ [1 Mark]

$\Rightarrow \theta = 30^o$ [1 Mark]

19. 14 cm [1 Mark]

20. K = 76 [1 Mark]

OR

Graphical representation of a frequency distribution may not be an ogive.
It may be a histogram. An ogive is a graphical representation of cumulative frequency distribution. [1 Mark]

21. Let $5-\sqrt{3}$ is not irrational.

Also $5-\sqrt{3} = \frac{p}{q}$ [p and q are integers, q $\neq$ 0] [½ Mark]

$\Rightarrow \frac{5q-p}{q} = \sqrt{3} \quad \Rightarrow \frac{5q-p}{q}$ is rational number. [1 Mark]

So, $\sqrt{3}$ should be rational.

But $\sqrt{3}$ is irrational, it cannot be equal to rational number, Hence $5-\sqrt{3}$ is irrational. [½ Mark]

22. Total no. of cards = 60 – 12 = 48
$\Rightarrow$ Total no. of outcomes = 48
Numbers are 13, 14, 15, 16, ..., 60.

(i) Numbers divisible by 5 are 15, 20, 25, 30, 35, 40, 45, 50, 55, 60. [½ Mark]
$\therefore$ Favourable outcomes = 10

$\therefore$ P(no. is divisible by 5) $= \frac{10}{48} = \frac{5}{24}$ [½ Mark]

(ii) Perfect square numbers are 16, 25, 36, 49 [½ Mark]
$\therefore$ Favourable outcomes = 4

$\therefore$ P(perfect square) $= \frac{4}{48} = \frac{1}{12}$ [½ Mark]

23. Since p, q, r are in A·P
$\therefore \; q-p = r-q$
$\Rightarrow 2q = r+p$ ... (i) [½ Mark]

$p^3 + r^3 - 8q^3 = (p^3+r^3) - (2q)^3 = (p+r)(p^2+r^2-rp) - (2q)^3$
$= (p+r)(p^2+r^2-rp) - (r+p)^3$ [From (i), 2q = $r+p$] [½ Mark]
$= (r+p)[(p^2+r^2-rp) - (r+p)^2] = (r+p)[p^2+r^2-rp-r^2-p^2-2pr]$
$= (r+p)(-3pr) = (2q)(-3pr) = -6pqr$ [From (i), $2q = r+p$] [1 Mark]

OR

The given AP is 18, $15\frac{1}{2}$, 13 ...., $-49\frac{1}{2}$.

Suppose the number of terms in given AP is n.
As, last term of an AP, l = a + (n – 1)d [½ Mark]

So, $-49\frac{1}{2} = 18 + (n-1)\left(15\frac{1}{2} - 18\right)$

$\Rightarrow -\frac{99}{2} = 18 + (n-1)\left(-\frac{5}{2}\right)$ [½ Mark]

$\Rightarrow \quad 99 = -36 + (n-1)5 \Rightarrow 5(n-1) = 99 + 36 = 135$

$\Rightarrow \quad n - 1 = \frac{135}{5} = 27 \Rightarrow n = 27 + 1 = 28$ [½ Mark]

Therefore, the number of terms in given AP is 28.

And, the sum of all 28 terms $= \frac{28}{2}\left(18 - 49\frac{1}{2}\right) = 14\left(\frac{36-99}{2}\right) = -441$ [½ Mark]

Hence, the number of terms in given AP is 28 and the sum of all its terms is – 441.

24. $x - 2y = 8$

$5x - 10y = c$

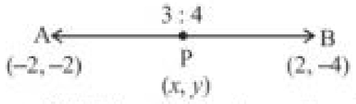
For unique solution

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [½ Mark]

here $\frac{a_1}{a_2} = \frac{1}{5}$ and $\frac{b_1}{b_2} = \frac{-2}{-10} = \frac{1}{5} \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2}$ [1 Mark]

Hence, the given statement is not true. [½ Mark]

25. $\because \quad AP = \frac{3}{7}AB$



[½ Mark]

$\because$ P divides AB in the ratio of 3:4

$\therefore$ AP : PB = 3 : 4

$\because \quad x = \frac{3(2) + 4(-2)}{3 + 4}$ and $y = \frac{3(-4) + 4(-2)}{3 + 4}$

So, $x = \frac{6 - 8}{7} = -\frac{2}{7}$ and $y = \frac{-12 - 8}{7} = -\frac{20}{7}$ [1 Mark]

Hence, coordinates of P are $\left(-\frac{2}{7}, -\frac{20}{7}\right)$ [½ Mark]

OR

Let A(7, 10), B(–2, 5) and C (3, –4) are vertices of triangle.

Then $AB = \sqrt{(7-(-2))^2 + (10-5)^2} = \sqrt{9^2 + 5^2} = \sqrt{81 + 25} = \sqrt{106}$ [½ Mark]

Now, $BC = \sqrt{(-2-3)^2 + (5-(-4))^2} = \sqrt{25 + 81} = \sqrt{106}$ [½ Mark]

$AC = \sqrt{(7-3)^2 + (10-(-4))^2} = \sqrt{4^2 + 14^2} = \sqrt{16 + 196} = \sqrt{212}$ [½ Mark]

As $AC^2 = AB^2 + BC^2$ and $AB = BC$

Hence, the points (7, 10), (–2, 5) and (3, –4) are the vertices of an isosceles right triangle. [½ Mark]

26. Number of 50 p coins = 100

Number of ₹ 1 coins = 50

Number of ₹ 2 coins = 20

Number of ₹ 5 coins = 10

Total number of coins = 180

$\Rightarrow$ Total no. of outcomes = 180 [½ Mark]

Then (i) P (50 p coin) = $\frac{100}{180} = \frac{5}{9}$

(ii) P (not a ₹ 5 coin) = 1 – P (a ₹ 5 coin) [½ Mark]

$$= 1 - \frac{10}{180} = 1 - \frac{1}{18} = \frac{17}{18}$$ [1 Mark]

27. Let a be any positive integer and b = 6

Then by Euclid's algorithm, a = bq + r, $0 \le r < b$ [½ Mark]

We have a = 6q + r ... (1)

Since $0 \le r < 6$, So, r = 0, 1, 2, 3, 4, 5

$\therefore$ From (1), for r = 0, a = 6q
for r = 1, a = 6q + 1
for r = 2, a = 6q + 2
for r = 3, a = 6q + 3
for r = 4, a = 6q + 4
for r = 5, a = 6q + 5 [½ Mark]

Since 6q is divisible by 2,

$\therefore$ 6q is even.

6q + 1 is not divisible by 2. So, 6q + 1 is odd.

6q + 2 is divisible by 2

$\therefore$ 6q + 2 is even. [1 Mark]

6q + 3 is not divisible by 2. So, 6q + 3 is odd.

6q + 4 is divisible by 2.

$\therefore$ 6q + 4 is even.

6q + 5 is not divisible by 2. So, 6q + 5 is odd. [½ Mark]

So, we see that 6q, 6q + 2, 6q + 4 are even. Since the number which are not divisible by 2 are odd integer. $\therefore$ 6q + 1, 6q + 3, 6q + 5 are odd integer. Hence any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer. [½ Mark]

28. Let (ax + b) be subtracted from given polynomial p(x), so that it is exactly divisible by $x^2 + x - 12$

Then, $q(x) = x^3 - 6x^2 - 15x + 80 - (ax + b) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$\because$ Dividend = Divisor × quotient + remainder [½ Mark]

But remainder will be zero.

$\therefore$ Dividend = Divisor × quotient

$q(x) = (x^2 + x - 12) \times$ quotient ...(i)

$\therefore$ $q(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$ ...(ii) [½ Mark]

After analysing eq. (i) and first two teams of RHS of eq. (ii), we get:

$(x - 7)(x^2 + x - 12) = x^3 + x^2 - 7x^2 - 12x - 7x + 84$

$= x^3 - 6x^2 - 19x + 84$ [1 Mark]

Hence, $x^3 - 6x^2 - 19x + 84 = x^3 - 6x^2 - (15 + a)x + (80 - b) - 15 - a = -19$

$\Rightarrow a = +4$ and $80 - b = 84 \Rightarrow b = -4$ [1 Mark]

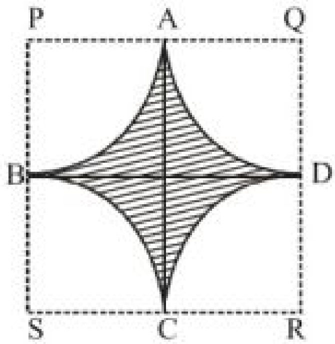
Hence, if in $p(x)$ we subtracted $4x - 4 = (ax + b)$, then it is exactly divisible by $x^2 + x - 12$

29. $m = a\cos\theta + b\sin\theta$
$(m)^2 = (a\cos\theta + b\sin\theta)^2$
$m^2 = a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta$ ...(i) [1 Mark]
also, $n = a\sin\theta - b\cos\theta$; $(n)^2 = (a\sin\theta - b\cos\theta)^2$
$n^2 = a^2\sin^2\theta + b^2\cos^2\theta - 2ab\cos\theta\sin\theta$ ...(ii) [1 Mark]
On adding equation (i) and (ii), we get
$m^2 + n^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$
$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$
$m^2 + n^2 = a^2 + b^2$ [1 Mark]
(Hence proved.)

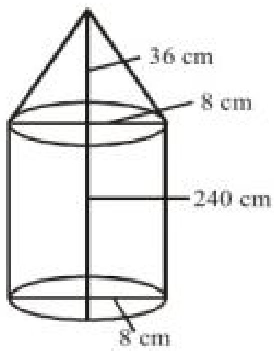
OR

$x = r\sin A\cos C,\ y = r\sin A\sin C$ and $z = r\cos A$
$x^2 + y^2 + z^2 = r^2\sin^2 A\cos^2 C + r^2\sin^2 A\sin^2 C + r^2\cos^2 A$ [1 Mark]
$= (r^2\sin^2 A)(\cos^2 C + \sin^2 C) + r^2\cos^2 A$ [1 Mark]
$= r^2\sin^2 A(1) + r^2\cos^2 A = r^2(\sin^2 A + \cos^2 A) = r^2$. **Hence Proved.** [1 Mark]

30. Draw square $PQRS$ as shown in the figure given below.
Here $PQ = QR = PS = RS = 14$ cm [1 Mark]
Area of the shaded portion = Area of square $PQRS$ – Area of four equal quadrants [1 Mark]
$= 14\times14 - 4\times\frac{1}{4}\pi\times(7)^2 = 196 - \frac{22}{7}\times7\times7 = 196 - 154 = 42\text{ cm}^2$. [1 Mark]



31. Volume of cylindrical part $= \pi(8)^2(240) = 64\pi\times240$ [½ Mark]
Volume of conical part $= \frac{1}{3}\pi\times(8)^2\times36 = (18)^2\pi(12)$ [½ Mark]



Total volume of iron $= 64\pi(240 + 12) = 64\times\frac{22}{7}\times252 = 64\times22\times36\text{ cm}^3$ [1 Mark]
$\therefore$ Total weight of pillar $= (64)(22)(36)\times7.8 = 395366.4$ gms $= 395.3664$ kg [1 Mark]

OR

Let radius of cylinder = r

$\therefore$ diameter of cylinder = 2r

$\therefore$ height of cylinder =2/3 (2r) $= \frac{4r}{3}$ [1 Mark]

Volume of cylinder $= \pi r^2 h = \pi r^2 \times \frac{4r}{3} = \frac{4\pi r^3}{3}$

Volume of sphere $= \frac{4}{3}\pi \times (4)^3 = \frac{256\pi}{3}$ [1 Mark]

Volume of cylinder = Volume of sphere $\Rightarrow \frac{4\pi r^3}{3} = \frac{256\pi}{3} \Rightarrow r^3 = \frac{256}{4} = 64 \Rightarrow r = 4$ cm. [1 Mark]

32. Let the numerator and denominator of the fraction be x and y respectively.

Then required fraction $= \frac{x}{y}$

$\therefore \quad y - x = 4$ ... (i)

and $y + 1 = 8(x-2) \quad \Rightarrow y + 1 = 8x - 16$ [1 Mark]

$\Rightarrow \quad y - 8x = -17$ ... (ii)

Subtracting (i) from (ii),

$y - 8x - (y - x) = -17 - 4 - 7x = -21$

$x = \frac{21}{7} = 3$

$\therefore \quad y = 4 + 3 = 7$ [1 Mark]

$\therefore \quad$ Required fraction $= \frac{3}{7} = \frac{a}{b}$.

Hence, $\frac{a+b}{2} = \frac{3+7}{2} = \frac{10}{2} = 5$ [1 Mark]

33. $\because$ Area of the triangle = 24 sq. units [½ Mark]

$\therefore \frac{1}{2}[1(2k+5) - 4(-5+1) - k(-1-2k)] = 24 \Rightarrow \frac{1}{2}[2k + 5 + 20 - 4 + k + 2k^2] = 24$ [½ Mark]

$\Rightarrow \quad 2k^2 + 3k + 21 = 48 \Rightarrow 2k^2 + 3k - 27 = 0 \Rightarrow (2k+9)(k-3) = 0$ [1 Mark]

Either $2k + 9 = 0$ or $k - 3 = 0$

$\Rightarrow \quad k = -\frac{9}{2}$, 3. [1 Mark]

OR

The point P (x, y) is equidistant form the points A (a + b, b – a) and B (a – b, a + b).

$\therefore \quad$ PA = PB

$\Rightarrow \sqrt{(a+b-x)^2 + (b-a-y)^2} = \sqrt{(a-b-x)^2 + (a+b-y)^2}$ [½ Mark]

$\Rightarrow (a+b-x)^2 + (b-a-y)^2 = (a-b-x)^2 + (a+b-y)^2$ [½ Mark]

$\Rightarrow \quad (a+b-x)^2 - (a-b-x)^2 = (a+b-y)^2 - (b-a-y)^2$ [½ Mark]

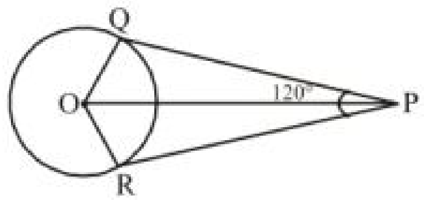
$\Rightarrow \quad (a+b-x+a-b-x)(a+b-x-a+b+x)$

$= (a+b-y+b-a-y)(a+b-y-b+a+y)$ [½ Mark]

$\Rightarrow \quad (2a-2x)(2b) = (2b-2y)(2a)$

$\Rightarrow \quad (a-x)b = (b-y)a \Rightarrow ab - bx = ab - ay \Rightarrow bx = ay$ [1 Mark]

34.



Given : PQ and PR are two tangents to the circle with centre O and $\angle QPR = 120°$

To Prove : $2PQ = PO$

Const : Join PO [1 Mark]

Proof : In ΔQPO and $\angle RPO$

Tangent is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OQP = \angle ORP = 90°$

$OQ = OR$ [Radius]

$OP = OP$ [Common side]

$\therefore \quad \Delta OQP \cong \Delta ORP$ [By SSA] [1 Mark]

$\Rightarrow \quad \angle QPO = \angle RPO = \frac{1}{2}\angle QPR$ [By CPCT]

$\angle QPO = \frac{1}{2}\angle QPR = 60°$

Now, $\cos 60° = \frac{PQ}{PO} \Rightarrow \frac{1}{2} = \frac{PQ}{PO}$

$PO = 2PQ$ **(Hence proved)** [1 Mark]

35. Let a, d be the first term and common difference of A.P.

Here, $\frac{S_m}{S_n} = \frac{m^2}{n^2} \Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$ [1 Mark]

$\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$ [1 Mark]

Here we can put m = 2m – 1 and n = 2n – 1, we get

$\frac{2a+(2m-1-1)d}{2a+(2n-1-1)d} = \frac{2m-1}{2n-1}$ [1 Mark]

$\frac{2[a+(m-1)d]}{2[a+(n-1)d]} = \frac{2m-1}{2n-1} \Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1}$

(Hence proved.) [1 Mark]

36. Since $XY \| AC$, we have

$\angle A = \angle BXY$ and $\angle C = \angle BYX$ [corres. $\angle$s] [½ Mark]

$\therefore \quad \Delta ABC \sim \Delta XBY$

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = \frac{AB^2}{XB^2}$ [1 Mark]

But, $ar(\Delta ABC) = 2 \times ar(\Delta XBY)$ [given]

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta XBY)} = 2$ [½ Mark]

From (i) and (ii), we get

$$\frac{AB^2}{XB^2} = 2 \Rightarrow \left(\frac{AB}{XB}\right)^2 = 2 \Rightarrow \frac{AB}{XB} = \sqrt{2} \Rightarrow AB = \sqrt{2}(XB)$$ [1 Mark]

$\Rightarrow \quad AB = \sqrt{2}(AB - AX) \Rightarrow \sqrt{2}AX = (\sqrt{2}-1)AB$ [½ Mark]

$\Rightarrow \quad \frac{AX}{AB} = \frac{(\sqrt{2}-1)}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$ [½ Mark]

Hence, $AX : AB = (2-\sqrt{2}) : 2$

OR

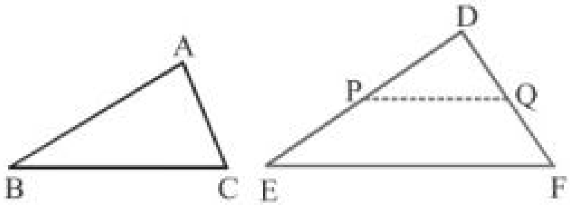
Given : Two ΔABC and ΔDEF such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To prove : $\Delta ABC \sim \Delta DEF$

Construction : Taking points P on DE and Q on DF such that $DP = AB$ and $DQ = AC$. Join PQ.[1Mark]

Proof : In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ (By construction)



Therefore, by converse of basic proportionality theorem,

$PQ \parallel EF$

So $\angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$ (corresponding angles)

Hence by AA similarity, $\Delta DPQ \sim \Delta DEF$ ... (i) [1 Mark]

Hence the corresponding sides of similar ΔDPQ and ΔDEF are proportional.

i.e., $\frac{DP}{DE} = \frac{PQ}{EF} \Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ $(\because DP = AB)$

But, $\frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{PQ}{EF} = \frac{BC}{EF} \Rightarrow PQ = BC$ ... (ii)

Now, in ΔABC and ΔDPQ

$AB = DP$ (By Construction)

$AC = DQ$ (By Construction)

$BC = PQ$ [From (ii)]

So by SSS congruence rule

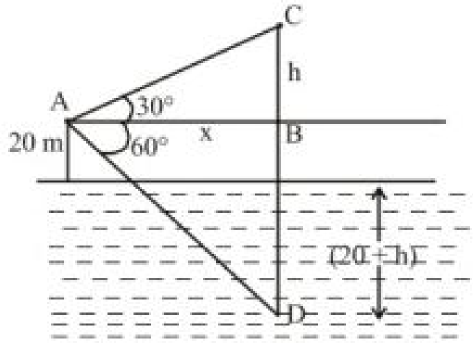
$\Delta ABC \cong \Delta DPQ$ ... (iii) [1 Mark]

From (i) and (iii)

$\Delta ABC \sim \Delta DPQ \sim \Delta DEF$ [1 Mark]

$\Rightarrow \Delta ABC \sim \Delta DEF$

37.



[1 Mark]

In ΔABC,

$\because \quad \frac{h}{x} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}\ h.$ [1 Mark]

Now, $\frac{40+h}{x} = \tan 60° = \sqrt{3} \Rightarrow x = \frac{40+h}{\sqrt{3}}$

So, $\sqrt{3}h = \frac{40+h}{\sqrt{3}} \Rightarrow h = 20\,m.$ [1 Mark]

Therefore, $x = 20\sqrt{3}$ m

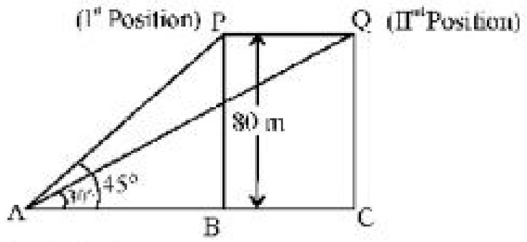
Hence, the distance of the cloud from A

i.e., $AC = \sqrt{(20)^2 + (20\sqrt{3})^2} = 40$ m [1 Mark]

OR

Suppose P and Q be two positions of the bird, and A be the point of observation.

Since, the angles of elevations of the bird for two positions P and Q from point A are 45° and 30° respectively.



[1 Mark]

So, $\angle PAB = 45°$ and $\angle QAB = 30°$

Now, BP = 80 m

For, ΔABP,

$\tan 45° = \frac{BP}{AB} \Rightarrow 1 = \frac{80}{AB} \Rightarrow AB = 80\,m$

For ΔACQ, $\tan 30° = \frac{CQ}{AC} \quad \frac{1}{\sqrt{3}} = \frac{80}{AC} \Rightarrow AC = 80\sqrt{3}m$ [1 Mark]

Therefore, $PQ = BC = AC - AB = 80(\sqrt{3}-1)\,m$ [1 Mark]

Then, the bird covers $80(\sqrt{3}-1)$ m in 2s.

Hence, speed of the bird $= \frac{80(\sqrt{3}-1)}{2}$ m/s $= 40(\sqrt{3}-1)$ m/s

$= 144(1.732-1)$ km/h $= 105.41$ km/h [1 Mark]

38. Put $x^2 - 5x = y$ then the given equation becomes [½ Mark]

$y^2 - 7y + 6 = 0 \Rightarrow y^2 - 6y - y + 6 = 0$

$\Rightarrow y(y-6) - 1(y-6) = 0 \Rightarrow (y-1)(y-6) = 0$

Either $y - 1 = 0 \Rightarrow y = 1$ or $y - 6 = 0 \Rightarrow y = 6$ [1 Mark]

then, putting the value of $y = 1$ in $x^2 - 5x$

$\Rightarrow x^2 - 5x - 1 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25 - 4\times 1(-1)}}{2\times 1}$

$\Rightarrow x = \frac{5 \pm \sqrt{25+4}}{2} \Rightarrow x = \frac{5 \pm \sqrt{29}}{2}$ [1 Mark]

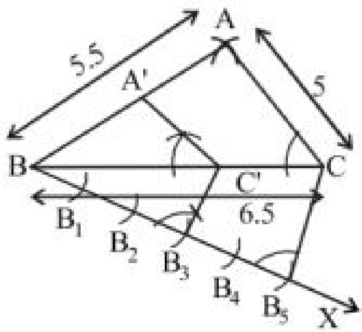
Putting the value of $y = 6$ in $x^2 - 5x$

$\Rightarrow x^2 - 5x - 6 = 0 \Rightarrow x^2 - 6x + x - 6 = 0$

$\Rightarrow x(x-6) + 1(x-6) = 0 \Rightarrow (x+1)(x-6) = 0$ [1 Mark]

Either $x + 1 = 0 \Rightarrow x = -1$ or $x - 6 = 0$

$\Rightarrow x = 6 \quad x = \left\{6, -1, \frac{5 \pm \sqrt{29}}{2}\right\}$ [½ Mark]

39. 

[3 Marks]

Steps of construction

1. Draw a line segment BC = 6.5 cm.
2. Take B as centre and mark an arc of length 5.5 cm above BC.
3. Similarly, take C as centre and cut an arc above BC of length 5 cm which cuts previous arc at point A.
4. Join AB and AC. The required triangle is formed i.e., ΔABC
5. Draw a ray BX below BC.
6. Mark equal arcs B_1, B_2, B_3, B_4 and B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
7. Join B_5C and draw a line || to B_5C from point B_3 which cuts BC at C′.
8. Draw a line || to AC from C′ which cuts AB at A′. [1 Mark]
9. Now, $\Delta A'BC'$ is the required triangle.

40.

Class interval	frequency f_i	x_i	$f_i x_i$
40 – 43	31	41.5	1286.5
43 – 46	58	44.5	2581
46 – 49	60	47.5	2850
49 – 52	K	50.5	50.5K
52 – 54	27	53.5	1444.5
	$\sum f_i = 176 + K$		$\sum f_i x_i = 8162 + 50.5K$

Mean $(\bar{x}) = 47.2 \Rightarrow \bar{x} = \frac{\sum f_i x_i}{\sum f_i} \Rightarrow 47.2 = \frac{8162 + 50.5K}{176 + K}$ [2 Marks]

$\Rightarrow 47.2(176 + K) = 8162 + 50.5K \Rightarrow 8307.2 + 47.2K = 8162 + 50.5K$

$\Rightarrow 8307.2 - 8162 = 50.5K - 47.2K \Rightarrow 145.2 = 3.3K$

$\Rightarrow K = \frac{145.2}{3.3} = 44$ [2 Marks]

There are 44 militants operating in the age group 49 – 52.

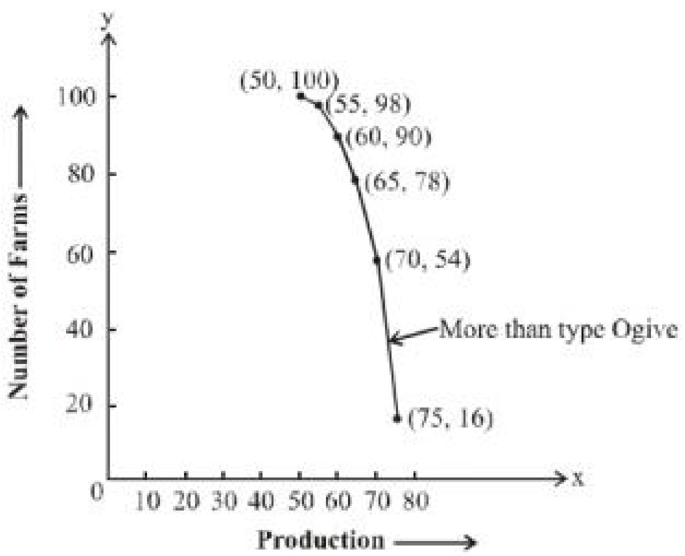
OR

More than type distribution

Production	Comulative frequency
More than or equal to 50	100
More than or equal to 55	98 (100 – 2)
More than or equal to 60	90 (98 – 8)
More than or equal to 65	78 (90 – 12)
More than or equal to 70	54 (78 – 24)
More than or equal to 75	16 (54 – 38)

[1 Mark]

Draw the points (50, 100), (55, 98), (60, 90), (65, 78), (70, 54) and (75, 16).



[3 Marks]