SAMPLE PAPER 5

1. **(b)** Let $x = 0.31783178......$ ... (i)

Multiply by 10000

$10000x = 3178.31783178........$ ... (ii)

Subtracting (i) from (ii)

$10000x = 3178.3178........$

$x = 0.3178........$

$-$ $-$

$999x = 3178$

$$x = \frac{3178}{999}$$

2. **(b)** $f(x) = x^6 + 1$

$f(1) = 1^6 + 1 = 2$

3. **(a)** Let Yash make x correct and y wrong answers then $3x - y = 40$ and $4x - 2y = 50$.

So, equations are $3x - y - 40 = 0$ ...(i)

and $4x - 2y - 50 = 0$

or $2x - y - 25 = 0$ ...(ii)

Subtracting (ii) from (i), $3x - 2x - 40 + 25 = 0$ or $x - 15 = 0$

$\Rightarrow x = 15$ and putting $x = 15$ in eq. (i)

$3 \times 15 - y - 40 = 0 \Rightarrow -y + 5 = 0$. So, $y = 5$

$\therefore$ Total number of questions in the test $= x + y = 15 + 5 = 20$

4. **(a)** α, β will satisfy the equation.

So, $\alpha^2 + \alpha\sqrt{\alpha} + \beta = 0$ ....(1)

and $\beta^2 + \beta\sqrt{\alpha} + \beta = 0$ ....(2)

Subtracting (2) from (1), we get

$a^2 - \beta^2 + \sqrt{\alpha}(\alpha - \beta) = 0$

$\Rightarrow (a - \beta)(\alpha + \beta + \sqrt{\alpha}) = 0$

$\Rightarrow \alpha = \beta$ or $\alpha + \beta + \sqrt{\alpha} = 0$ ....(3)

Also by (2) $\beta(\beta + \sqrt{\alpha} + 1) = 0$

$\Rightarrow \beta = 0$ or $1 + \sqrt{\alpha} + \beta = 0$ ....(4)

Solving (3) an d(4) we get $\alpha = 1$ and by (4) $\beta = -2$.

$\therefore \alpha^2 - \beta^2 = 1 - 4 = -3$

5. **(c)**

Class	f_i	x_i	$f_i x_i$
0 – 20	17	10	170
20 – 40	f_1	30	$30f_1$
40 – 60	32	50	1600
60 – 80	f_2	70	$70f_2$
80 – 100	19	90	1710
	$\Sigma f_i = 68 + f_1 + f_2$		$\Sigma f_i x_i = 3480 + 30f_1 + 70f_2$

We have $\Sigma f_i = 120$

$\Rightarrow \quad 68 + f_1 + f_2 = 120 \Rightarrow f_1 + f_2 = 52$ …(i)

Now, mean = 50

$$\Rightarrow \quad 50 = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 50 = \frac{3480 + 30f_i + 70f_2}{120}$$

$\Rightarrow 6000 = 3480 + 30f_1 + 70f_2 \Rightarrow 30f_1 + 70f_2 = 2520$ ...(ii)

Solving (i) and (ii) we get; $f_1 = 28, f_2 = 24$

6. **(b)** We have, $S = \frac{n}{2}(a + l) \Rightarrow \frac{2S}{a + l} = n$ ...(i)

Also, $l = a + (n-1)\,d \Rightarrow d = \frac{l - a}{n - 1} = \frac{l - a}{\frac{2S}{a + l} - 1} = \frac{l^2 - a^2}{2S - (l + a)}$

$\therefore \quad k = 2S \left[\text{From (i) } n = \frac{2S}{a + l}\right]$

7. **(a)** Let the number of blue balls = x

$\therefore$ Total number of balls = 5 + x

P (blue ball) = $\frac{x}{5 + x}$; P (red ball) = $\frac{5}{5 + x}$

Given that P (blue) = 2 × P (red)

$$\frac{x}{5 + x} = 2 \times \frac{5}{5 + x} \Rightarrow \frac{x}{5 + x} = \frac{10}{5 + x}$$

On solving we get $x = 10$

8. **(d)** All the given identities are correct.

9. **(a)** Let present age of Nuri = x years

Let present age of Sonu = y years

Five years ago,

$x - 5 = 3(y - 5)$

$x - 5 = 3y - 15$

$x - 3y = -10$ ...(i)

Ten years later, $(x + 10) = 2(y + 10)$

$x + 10 = 2y + 20$

$x - 2y = 10$ ...(ii)

Subtracting (ii) from (i), we get : $-y = -20$

$\Rightarrow \quad y = 20$
Substituting y = 20 in (ii), we get : $x - 2 \times 20 = 10$
$\Rightarrow \quad x = 50$
So, present age of Nuri is 50 years and present age of Sonu is 20 years

10. **(c)** A_3

11. $\sin A + \operatorname{cosec} A = 3$

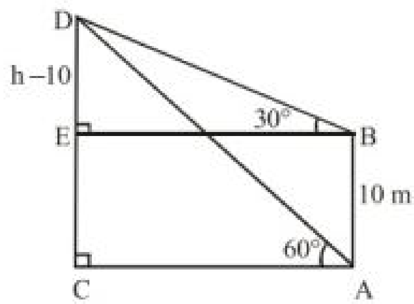
$\Rightarrow \sin A + \frac{1}{\sin A} = 3 \Rightarrow \frac{\sin^2 A + 1}{\sin A} = 3$ or $\sin^2 A + 1 = 3 \sin \mathbf{A}$

Squaring both sides, we get

$1 + \sin^4 A + 2\sin^2 A = 9\sin^2 A \Rightarrow 1 + \sin^4 A = 7\sin^2 A \Rightarrow \frac{1 + \sin^4 A}{\sin^2 A} = 7$

Answer: 7

12. Let AB and CD be the height of the pole and the tower respectively.



Let CD = h
Then $\angle DAC = 60°$ and $\angle DBE = 30°$

Now $\frac{CD}{CA} = \tan 60° = \sqrt{3}$

$\therefore \ CD = \sqrt{3}\ CA \Rightarrow \frac{h}{\sqrt{3}} = CA$

Again $\frac{DE}{BE} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore (h - 10) = \frac{BE}{\sqrt{3}} = \frac{CA}{\sqrt{3}} = \frac{h/\sqrt{3}}{\sqrt{3}} = \frac{h}{3}$ $[\because BE = CA]$

$\Rightarrow 3h - 30 = h \Rightarrow 2h = 30 \Rightarrow h = 15$
Hence, the height of the tower is 15 m
Answer: 15 m

13. $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow ar(\Delta ABC) = \left(\frac{2.1}{2.8}\right)^2 \times ar(\Delta DEF) = 9\ cm^2$

Answer: 9 cm^2

OR

Since $\Delta ABC \sim \Delta DEF$

$\therefore \ \frac{AB}{DE} = \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF}$

$\Rightarrow \frac{4}{6} = \frac{\text{Perimeter of } \Delta ABC}{DE + EF + FD}$

$\Rightarrow \quad \frac{2}{3} = \frac{\text{Perimeter of } \Delta ABC}{6 + 9 + 12}$

$\Rightarrow$ Perimeter of Δ ABC

$= \frac{2 \times 27}{3} = 2 \times 9 = 18$ cm

Answer: 18 cm

14. Combined mean $= \dfrac{9\times100+6\times80}{15} = 92$

Answer: 92

15. Here, $\sec\theta + \operatorname{cosec}\theta = b$

$$\frac{1}{\cos\theta}+\frac{1}{\sin\theta} = b \Rightarrow \frac{\sin\theta+\cos\theta}{\cos\theta\sin\theta} = b \Rightarrow \frac{a}{\cos\theta\sin\theta} = b \Rightarrow \cos\theta\sin\theta = \frac{a}{b} \qquad ...(i)$$

also, $\sin\theta + \cos\theta = a \Rightarrow (\sin\theta + \cos\theta)^2 = a^2 \Rightarrow (1 + 2\sin\theta\cos\theta) = a^2 \Rightarrow 2\sin\theta\cos\theta = a^2 - 1$

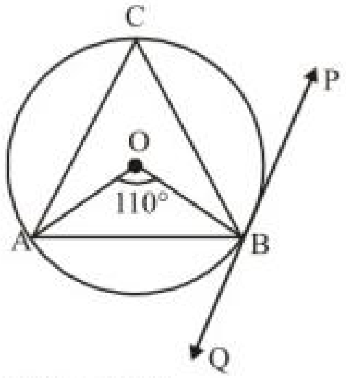
$\dfrac{2a}{b} = a^2 - 1$ [Using eq^n (i)]

$\Rightarrow \dfrac{2a}{b}\times b = b(a^2-1) \Rightarrow 2a = b(a^2-1)$

Answer: 2a

16. In ΔOAB, OA = OB (radii of the circle)

$\therefore \angle OAB = \angle OBA$



In ΔOAB, $\angle AOB + \angle OAB + \angle OBA = 180°$

$2\angle OBA = 180° - 110°$

$\Rightarrow \angle OBA = 35°$

Since BQ is a tangent at B

$\therefore \angle OBQ = 90°$

$\Rightarrow \angle OBA + \angle ABQ = 90° \Rightarrow 35° + \angle ABQ = 90°$

$\Rightarrow \angle ABQ = 90° - 35° = 55°$ [1 Mark]

17. Percentage of bad mangoes

= 100% – percentage of good mangoes = 100% – 90% = 10%

$\therefore$ probability of bad magoes $= \dfrac{10}{100} = \dfrac{1}{10}$ [1 Mark]

18. Since, $PQ = QR \Rightarrow Q$ is mid–point of PR.

$\therefore$ Using mid–point formula,

$1 = \dfrac{6+x}{2} \Rightarrow 6 + x = 2 \Rightarrow x = -4.$ [1 Mark]

OR

$x = 1, y = 1$ [1 Mark]

19. PB = 2cm [1 Mark]

20. $\tan 2\theta = \cot(\theta + 18°)$

$\Rightarrow \cot(90 - 2\theta) = \cot(\theta + 18°)$

$\Rightarrow 90-2\theta=\theta+18^\circ$
$\Rightarrow 3\theta=90^\circ-18^\circ$
$\Rightarrow 3\theta=72^\circ \Rightarrow \theta=24^\circ$ [1 Mark]

21. Largest possible amount of cheque will be the HCF (6075, 8505).
Applying Euclid's division lemma to 8505 and 6075, we have,
$8505=6075\times1+2430$ [½ Mark]
Since, remainder $2430 \neq 0$ again applying division lemma to 6075 and 2430
$6075=2430\times2+1215$ [½ Mark]
Again remainder $1215 \neq 0$
So, again applying the division lemma to 2430 and 1215
$2430=1215\times2+0$ [½ Mark]
Here the remainder is zero
So, H.C.F = 1215 [½ Mark]
Therefore, the largest possible amount of each cheque will be 1215.

22. Series : $70+68+66+\ldots\ldots\ldots\ldots+40$
Here $a=70, d=68-70=-2, t_n=\ell=40$.
Now, $t_n=a+(n-1)d=70+(n-1)(-2)\Rightarrow 40=70-2n+2\Rightarrow n=16$ [1 Mark]

$\therefore\ S_n=\frac{n}{2}[a+\ell] \Rightarrow S_n=\frac{16}{2}[70+40]=880$ [1 Mark]

23. $S=\{S, M, T, W, Th, F, Sa\}\Rightarrow n(S)=7$ [½ Mark]
A non-leap year contains 365 days, i.e., 52 weeks + 1 day.
$E=\{Sa\}, n(E)=1$ [½ Mark]

$\therefore\ P(E)=\frac{n(E)}{n(S)}=\frac{1}{7}$ [1 Mark]

24. Let there be x blue, y green and z white marbles in the jar.
Then, $x+y+z=54$ ...(i) [½ Mark]

$\therefore\ P(\text{selecting a blue marble})=\frac{x}{54} \Rightarrow \frac{x}{54}=\frac{1}{3}\Rightarrow x=18$ [½ Mark]

Similarly, $P(\text{selecting a green marble})=\frac{y}{54} \Rightarrow \frac{y}{54}=\frac{4}{9}\Rightarrow y=24$ [½ Mark]

Substituting the values of x and y in (i), $z=12$
Hence, the jar contains 12 white marbles. [½ Mark]

25. Given lines, $3x+2ky-2=0$ and $2x+5y-1=0$
Here, $a_1=3, b_1=2k, c_1=-2, a_2=2, b_2=5, c_2=-1$ [1 Mark]

Condition for parallel lines is $\frac{3}{2}=\frac{2k}{5} \Rightarrow k=\frac{15}{4}$ [1 Mark]

OR

Let x be the no. of coins of ₹1 and y be the no. of coins of ₹ 2 According to the question,
$x+y=50$ ...(1)
and $x+2y=75$ ...(2) [½ Mark]
from (1) and (2)
$y=25$ [½ Mark]
and $x=25$ [½ Mark]

Number of coins of ₹ 1 = 25
Number of coins of ₹ 2 = 25 [½ Mark]

26. Since points are collinear,
so, $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$ [½ Mark]
Taking (7, –2) as (x_1, y_1), (5, 1) as (x_2, y_2) and (3, k) as (x_3, y_3), we have
$7(1 - k) + 5(k + 2) + 3(-2 - 1) = 0$ [½ Mark]
or $7 - 7k + 5k + 10 - 9 = 0 \Rightarrow 2k = 8 \Rightarrow k = 4.$ [1 Mark]

OR

Given that
Distance between A(–3, –14) and B(a, –5), AB = 9
∴ using distance formula

$$\sqrt{(a+3)^2 + (-5+14)^2} = 9$$ [½ Mark]

$$\Rightarrow \sqrt{(a+3)^2 + (9)^2} = 9$$ [½ Mark]

On squaring both the sides,
$(a+3)^2 + 81 = 81$ [½ Mark]
$\Rightarrow (a+3)^2 = 0$
$\Rightarrow a = -3$ [½ Mark]

27. eqn. $a^2 - b^2 = (a - b)(a + b)$ .....(i)
$\because$ $a^2 - b^2$ is a prime number [1 Mark]
$\therefore$ One of the two factors = 1
So, $a - b = 1$ $[\because a - b < a + b]$
$\because$ The only divisors of a prime number are 1 and itself. [1 Mark]
(i) become $a^2 - b^2 = 1\,(a + b)$
or $a^2 - b^2 = a + b$
e.g., $3^2 - 2^2 = 5$ (which is prime)
$\Rightarrow 3^2 - 2^2 = 3 + 2$ [1 Mark]

28. [2 Marks]

$$\begin{array}{r|l} 3x^2+4x+1 \;) & 6x^4 + 8x^3 + 17x^2 + 21x + 7 \;(\; 2x^2 + 5 \\ & \underline{6x^4 + 8x^3 + 2x^2} \\ & \quad -\quad -\quad - \\ & 15x^2 + 21x + 7 \\ & \underline{15x^2 + 20x + 5} \\ & \quad -\quad -\quad - \\ & x + 2 \end{array}$$

$\therefore$ $ax + b = x + 2$ [½ Mark]
On comparing both sides, $a = 1$ and $b = 2$. [½ Mark]

29. $m^2 - n^2 = (m + n) \times (m - n)$
$= \{(\tan\theta + \sin\theta) + (\tan\theta - \sin\theta)\} \times \{(\tan\theta + \sin\theta) - (\tan\theta - \sin\theta)\}$ [½ Mark]
$= \{2\tan\theta\} \times \{2\sin\theta\} = 4\tan\theta\sin\theta$ [½ Mark]

$$= 4\sqrt{\tan^2\theta\sin^2\theta} = 4\sqrt{(\sec^2\theta - 1)\sin^2\theta}$$ [½ Mark]

$$= 4\sqrt{\sec^2\theta\sin^2\theta - \sin^2\theta} \qquad \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$ [½ Mark]

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)} = 4\sqrt{mn}$$ [1 Mark]

OR

$$\frac{\cos^2\theta}{\cot^2\theta-\cos^2\theta}=3 \Rightarrow \frac{\cos^2\theta}{\frac{\cos^2\theta}{\sin^2\theta}-\cos^2\theta}=3$$ [½ Mark]

$$\Rightarrow \frac{\cos^2\theta\times\sin^2\theta}{\cos^2\theta-\sin^2\theta\cos^2\theta}=3 \Rightarrow \frac{\sin^2\theta\cos^2\theta}{\cos^2\theta(1-\sin^2\theta)}=3$$ [1Mark]

$$\Rightarrow \frac{\sin^2\theta}{\cos^2\theta}=3 \Rightarrow \tan^2\theta=3 \Rightarrow \tan\theta=\sqrt{3}$$ [1 Mark]

$\tan\theta=\tan 60^o \Rightarrow \theta=60^o$ (acute angle) [½ Mark]

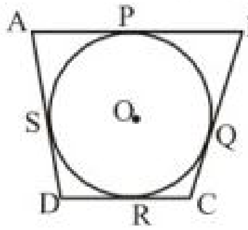
30.

Number of pages	x_i	f_i	$f_i x_i$
15.5-18.5	17	1	17
18.5-21.5	20	3	60
21.5-24.5	23	4	92
24.5-27.5	26	9	234
27.5-30.5	29	13	377
		$\sum f_i=30$	$\sum f_i x_i=780$

[2 Marks]

Mean $(\bar{x})=\dfrac{\sum f_i x_i}{\sum f_i} \Rightarrow \bar{x}=\dfrac{780}{30}$, $\bar{x}=26$ [1 Mark]

31. Let there be a circle with centre O whereas AB, BC, CD and DA are tangents at P, Q, R and S respectively.



[1 Mark]

Here,

AP = AS ...(i)

BP = BQ ...(ii)

CR = CQ ...(iii)

DR = DS ..(iv)

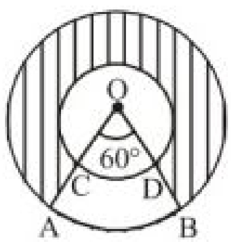
[Tangents drawn from a point (outside the circle) on a given circle are equal in lengths]

From equations (i), (ii), (iii) and (iv), [1 Mark]

(AP + BP) + (CR + DR) = (BQ + CQ) + (DS + AS)

AB + CD = BC + DA (Hence proved.) [1 Mark]

32.



Radius of inner circle, r = 21 cm
Radius of outer circle, R = 42 cm
Area of (ABCD) = Area of sector (OAB) – Area of sector (OCD)

$$= \frac{60}{360} \times \pi\left(42^2 - 21^2\right)$$ [1 Mark]

$$= \frac{1}{6} \times \frac{22}{7} \times (42+21)(42-21) = \frac{1}{6} \times \frac{22}{7} \times 63 \times 21 = 11 \times 21 \times 3 = 693 \text{ cm}^2$$ [1 Mark]

Area of shaded region = Area of outer circle – Area of inner circle – Area (ABCD)

$$= \pi(42)^2 - \pi(21)^2 - 693 = \frac{22}{7} \times 63 \times 21 - 693 = 4158 - 693 = 3465 \text{ cm}^2$$ [1 Mark]

33. Suppose the radii of the two spheres are r_1 and r_2 respectively.
Given, that,

ratio of volumes $(V_1 : V_2) = 64 : 27 \Rightarrow \dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{64}{27} \Rightarrow \dfrac{r_1^3}{r_2^3} = \dfrac{64}{27} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{4}{3}$ [2 Marks]

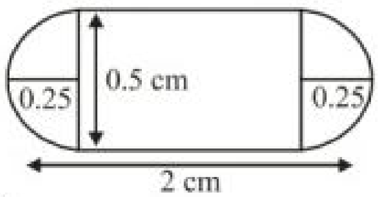
Now, ratio of surface areas $= \dfrac{4\pi r_1^2}{4\pi r_2^2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{4}{3}\right)^2 = 16 : 9$ [1 Mark]

Hence, the required ratio of their surface areas is 16 : 9.

OR

Since, diameter of cylinder = diameter of hemisphere = 0.5 cm

$\therefore$ Radius of cylinder (r) = radius of hemisphere (r) $= \dfrac{0.5}{2} = 0.25$ cm [½ Mark]



and total length of capsule = 2 cm
$\therefore$ Length of cylindrical part of capsule,
h = 2 – (0.25 + 0.25) = 1.5 cm [½ Mark]
Now, volume of capsule= Volume of cylindrical part + 2 × Volume of hemisphere

$$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3 = \frac{22}{7}\left[(0.25)^2 \times 1.5 + \frac{4}{3} \times (0.25)^3\right]$$ [1 Mark]

$$= \frac{22}{7}[0.09375 + 0.0208] = \frac{22}{7} \times 0.11455 = 0.36 \text{ cm}^3$$ [1 Mark]

Hence the volume of capsule is 0.36 cm^3

34. Since, A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.
So, the slope of AB and BC will be same. [1 Mark]
Slope of AB = Slope of BC

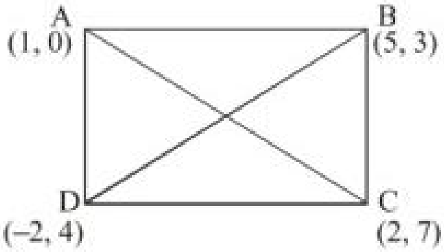
$$\Rightarrow \frac{2k+3-2k}{3k-(k+1)} = \frac{5k-(2k+3)}{(5k-1)-3k} \Rightarrow \frac{3}{2k-1} = \frac{3k-3}{2k-1}$$ [1 Mark]

$\Rightarrow 3(2k-1) = (2k-1)(3k-3) \Rightarrow (2k-1)(3k-3) - 3(2k-1) = 0$

$\Rightarrow (2k-1)(3k-3-3) = 0 \Rightarrow (2k-1)(3k-6) = 0 \Rightarrow k = \frac{1}{2}, 2$ [1 Mark]

So, the value of k is 2 or $\frac{1}{2}$.

OR



Coordinates of the mid-point of diagonal AC = $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$ [1 Mark]

Coordinates of the mid-point of diagonal BD = $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$ [1 Mark]

Since, the coordinates of the mid-points of diagonals AC and BD are same.
$\therefore$ They bisect each other.
Hence, ABCD is a parallelogram. [1 Mark]

35. Let the usual speed of plane = x km/hr

Time taken by plane to cover distance = $\frac{1500}{x}$ hrs. ... (i) [½ Mark]

But when speed is increased by 250 km/hr then, increased speed = (x + 250) km/hr [½ Mark]

$\therefore$ Time taken to cover distance by the plane = $\frac{1500}{x+250}$ hrs. ......(ii) [½ Mark]

According to the question, $\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$ [½ Mark]

$\Rightarrow 1500\left[\frac{1}{x} - \frac{1}{x+250}\right] = \frac{1}{2} \Rightarrow 1500\left[\frac{x+250-x}{x(x+250)}\right] = \frac{1}{2} \Rightarrow 1500\left[\frac{250}{x^2+250x}\right] = \frac{1}{2}$ [½ Mark]

$\Rightarrow 750000 = x^2 + 250x \Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0 \Rightarrow x(x+1000) - 750(x+1000) = 0$

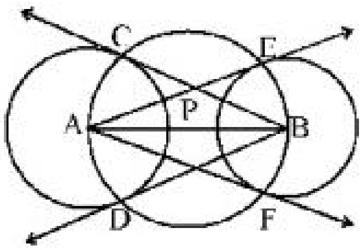
$\Rightarrow (x-750)(x+1000) = 0$ [1 Mark]

Either $x + 1000 = 0 \Rightarrow x = -1000$ (rejected) since speed is never negative

or $x - 750 = 0 \Rightarrow x = 750$ [½ Mark]

Usual speed of plane = 750 km/hr

36.



[3 Marks]

Steps of construction :

(i) Draw a line AB of length 8 cm.

(ii) With centre A and radius 4 cm, draw a circle, which intersect AB at point P.

(iii) With centre B and radius 3 cm, draw the second circle.

(iv) With centre P and radius PA or PB draw third circle, which intersects the circle drawn in step (ii) at C & D and the circle drawn in step (iii) at E & F.

(v) Draw rays AE, AF, BC and BD.

(vi) Thus AE, AF, BC and BD are required tangents. [1 Mark]

37. We know that $\ell = \sqrt{r^2 + h^2}$, $v = \frac{1}{3}\pi r^2 h$, $c = \pi r\ell$ [1 Mark]

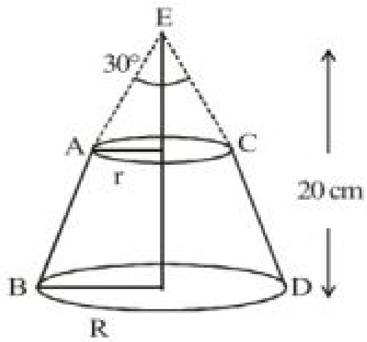
LHS. $= 3\pi vh^3 - c^2h^2 + 9v^2 = 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^3 - (\pi r\ell)^2 h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$ [1 Mark]

$= 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^3 - \pi^2 r^2\left(r^2 + h^2\right) \times h^2 + 9 \times \frac{1}{9}\pi^2 r^4 h^2 \qquad \left(\because \ \ell = \sqrt{h^2 + r^2}\right)$ [1 Mark]

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0 =$ **RHS** [1 Mark]

OR

From the figure,



$\frac{R}{20} = \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow R = \frac{20}{\sqrt{3}}$ cm [1 Mark]

$\frac{r}{10} = \tan 30° = \frac{1}{\sqrt{3}} \qquad \Rightarrow r = \frac{10}{\sqrt{3}}$ cm

$h = 10$ cm is the height of the frustum.

Volume of the metal in the frustum ACDB $= \frac{1}{3}\pi \times h \times \{R^2 + r^2 + Rr\}$

$= \frac{1}{3}\pi \times 10 \times \left\{\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right\} \text{cm}^3 = \frac{7000}{9}\pi \text{ cm}^3$ [1 Mark]

Let the length of wire be x.

Now, suppose wire of diameter $\frac{1}{16}$ cm is made of length x cm.

$\therefore$ volume of wire = volume of frustum. [1 Mark]

$\Rightarrow \pi \times \left(\frac{1}{32}\right)^2 \times x = \frac{7000}{9}\pi \qquad \left[\because \ r = \frac{d}{2}\right]$

$x = \frac{71680}{9}$ m $= 7964.4$ m $\qquad \left[\because \ 1 \text{ cm} = \frac{1}{100} \text{m}\right]$

Hence, required length of wire is 7964.4 cm [1 Mark]

38. Since, $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{a}+\frac{1}{c}\right)$ and $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

So, $2b\left(\frac{1}{a}+\frac{1}{c}\right) = a\left(\frac{1}{b}+\frac{1}{c}\right)+c\left(\frac{1}{a}+\frac{1}{b}\right)$ [1 Mark]

$\Rightarrow \frac{2b}{a}+\frac{2b}{c}=\frac{a}{b}+\frac{a}{c}+\frac{c}{a}+\frac{c}{b} \Rightarrow \frac{2b}{a}-\frac{c}{a}+\frac{2b}{c}-\frac{a}{c}=\frac{a}{b}+\frac{c}{b} \Rightarrow \frac{2b-c}{a}+\frac{2b-a}{c}=\frac{a+c}{b}$

$\Rightarrow [2bc-c^2+2ab-a^2]\,b=[a+b]\,ac \Rightarrow 2b^2c-b^2c+2ab^2-a^2b=a^2c+ac^2$

$\Rightarrow 2b^2(a+c)=a^2(b+c)+c^2(a+b)$ [½ Mark]

$\Rightarrow 2b^2(a+c)=a^2c+c^2a+a^2b+c^2b$ [½ Mark]

$\Rightarrow 2b^2(a+c)=ac(a+c)+b(a^2+c^2+2ac-2ac)$ [½ Mark]

$\Rightarrow 2b^2(a+c)+2abc=ac(a+c)+b(a+c)^2$ [½ Mark]

$\Rightarrow 2b(ab+bc+ca)=(a+c)(ab+bc+ca)$ [½ Mark]

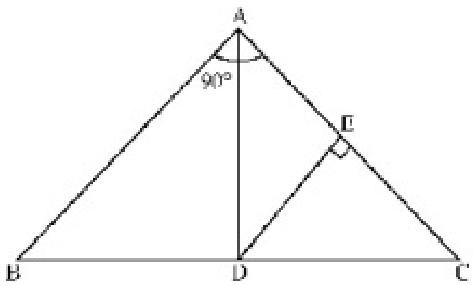
$\Rightarrow 2b=a+c \; (\because ab+bc+ca\neq 0)$

So, a, b, c are in A.P. (Hence proved.) [½ Mark]

39. It is give that AD is the bisector of $\angle A$ of ΔABC.

$\therefore \frac{AB}{AC}=\frac{BD}{DC} \Rightarrow \frac{AB}{AC}+1=\frac{BD}{DC}+1$ [Adding 1 on both sides] [½ Mark]

$\Rightarrow \frac{AB+AC}{AC}=\frac{BD+DC}{DC}$ [½ Mark]



$\Rightarrow \frac{AB+AC}{AC}=\frac{BC}{DC}$ ...(1) [½ Mark]

In Δ's CDE and CBA, we have

$\angle DCE=\angle BCA=\angle C$ [Common]

$\angle BAC=\angle DEC$ [Each equal to 90°]

So, by AA-criterion of similarity, we have [1 Mark]

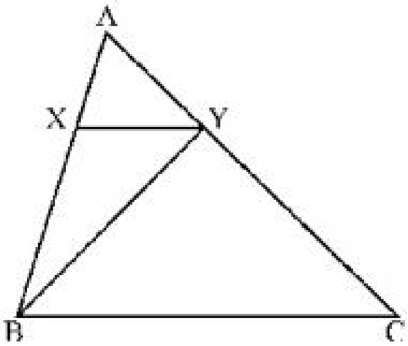
$\Delta CDE \sim \Delta CBA \Rightarrow \frac{CD}{CB}=\frac{DE}{BA} \Rightarrow \frac{AB}{DE}=\frac{BC}{DC}$ ...(2) [½ Mark]

From (1) and (2), we have $\frac{AB+AC}{AC}=\frac{AB}{DE}$

$\Rightarrow DE\times(AB+AC)=AB\times AC$ [1 Mark]

OR

$\Delta AXY \sim \Delta ABC$ [By AA criterion of similarity] [1 Mark]



$\therefore \quad \dfrac{ar(\Delta AXY)}{ar(\Delta ABC)} = \dfrac{AX^2}{AB^2}$ [½ Mark]

[The areas of two similar Δs are in the ratio of the squares of the corresponding sides]

Now, AX : XB = 3 : 5

$\therefore \quad \dfrac{AX}{AB} = \dfrac{AX}{AX+XB} = \dfrac{3}{8}$ [½ Mark]

$\therefore \quad \dfrac{ar(\Delta AXY)}{ar(\Delta ABC)} = \dfrac{AX^2}{AB^2} = \left(\dfrac{AX}{AB}\right)^2 = \left(\dfrac{3}{8}\right)^2 = \dfrac{9}{64}.$ [1 Mark]

$\Rightarrow \quad ar(\Delta AXY)\ \dfrac{9}{64} \times ar(\Delta ABC) = \dfrac{9}{64} \times 16 = \dfrac{9}{4}\ cm^2$

$\therefore \quad \dfrac{ar(\Delta AXY)}{ar(\Delta BXY)} = \dfrac{AX}{BX}$ [∵ Height of two ΔS AXY and BXY is same]

$\therefore \quad ar(\Delta BXY) = \dfrac{5}{3} \times ar(\Delta AXY)$ [1 Mark]

$= \dfrac{5}{3} \times \dfrac{9}{4} = \dfrac{15}{4} = 3.75\ cm^2.$

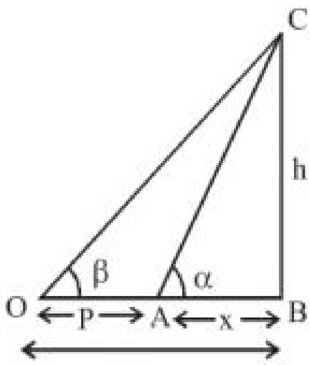
40. Let BC be the tower

Given, BC = h

Let O and A be the two objects. [½ Mark]

Given OA = P

In right Δ ABC, $\tan\alpha = \dfrac{h}{x} \Rightarrow x = \dfrac{h}{\tan\alpha}$ ....(1) [1 Mark]



[½ Mark]

In right Δ OBC, $\tan\beta = \dfrac{h}{P+x} \Rightarrow \tan\beta = \dfrac{h}{P+\left(\dfrac{h}{\tan\alpha}\right)}$ [using (1)] [1 Mark]

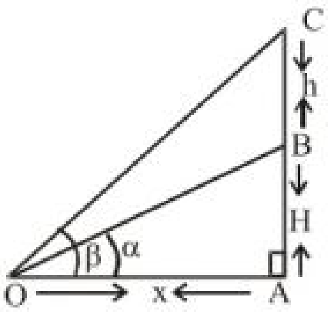
$$\Rightarrow p\tan\beta + \frac{h\tan P}{\tan\alpha} = h \Rightarrow P\tan\beta = h\left(1-\frac{\tan\beta}{\tan\alpha}\right)$$

$\Rightarrow P = \dfrac{h(\tan\alpha - \tan\beta)}{\tan\beta\tan\alpha} \Rightarrow h = \dfrac{P\ \tan\beta\tan\alpha}{\tan\alpha - \tan\beta}$ [1 Mark]

OR

Let AB be the tower and BC be the flag.
Let AB = H, BC = h

In right Δ OAB, $\cot\alpha = \dfrac{x}{H} \Rightarrow x = H\cot x$ ...(1)



[1 Mark]

and in right Δ OCA,

$\cot\ \beta = \dfrac{x}{H+h} \Rightarrow x = (H+h)\ \cot\beta$ ...(2) [1 Mark]

Form (1) & (2)

$H\cot\alpha = (H+h)\cot\beta$

$\Rightarrow H(\cot\alpha - \cot\beta) = h\cot\beta$ [1 Mark]

$$\Rightarrow H = \frac{h\ \cot\beta}{\cot\alpha - \cot\beta} = \frac{h}{\tan\beta\left(\dfrac{1}{\tan\alpha} - \dfrac{1}{\tan\beta}\right)} = \frac{h}{\tan\beta\left(\dfrac{\tan\beta - \tan\alpha}{\tan\alpha.\tan\beta}\right)}$$

$H = \dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$ (Hence Proved.) [1 Mark]